Solutions for In-Class Problems 19 for Monday, April 9

These problems are from Pre-Class Problems 19.

1. Solve the following systems of equations using Gaussian elimination.

a. $$\begin{aligned} 3x - 3y - 6z &= 13 \\ -5x + 15y + 25z &= -35 \\ 6x - 12y - 21z &= 34 \end{aligned}$$

b. $$\begin{aligned} 2x + 5y + 3z &= -8 \\ 4x + 11y + 2z &= -14 \\ -6x - 14y - 13z &= 22 \end{aligned}$$

c. $$\begin{aligned} x - 2y + 5z &= 9 \\ -4x + 7y - 23z &= -43 \\ 3x - 5y + 18z &= 34 \end{aligned}$$

d. $$\begin{aligned} 4x - 2y + z &= 12 \\ -3x + 4y - 5z &= -13 \\ 9x - 2y - 2z &= 23 \end{aligned}$$

SOLUTIONS:

1a. $$\begin{aligned} 3x - 3y - 6z &= 13 \\ -5x + 15y + 25z &= -35 \\ 6x - 12y - 21z &= 34 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} 3 & -3 & -6 & 13 \\ -5 & 15 & 25 & -35 \\ 6 & -12 & -21 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -3 & -6 & 13 \\ -5 & 15 & 25 & -35 \\ 6 & -12 & -21 & 34 \end{bmatrix} \xrightarrow{-\frac{1}{5}R_1} \begin{bmatrix} 3 & -3 & -6 & 13 \\ 1 & -3 & -5 & 7 \\ 6 & -12 & -21 & 34 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\begin{bmatrix} 1 & -3 & -5 & 7 \\ 3 & -3 & -6 & 13 \\ 6 & -12 & -21 & 34 \end{bmatrix} \xrightarrow[-6R_1 + R_3]{-3R_1 + R_2} \begin{bmatrix} 1 & -3 & -5 & 7 \\ 0 & 6 & 9 & -8 \\ 0 & 6 & 9 & -8 \end{bmatrix} \xrightarrow{-R_1 + R_3}$$

$$\begin{bmatrix} 1 & -3 & -5 & 7 \\ 0 & 6 & 9 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{6}R_1} \begin{bmatrix} 1 & -3 & -5 & 7 \\ 0 & 1 & \frac{3}{2} & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 2 reads $y + \dfrac{3}{2}z = -\dfrac{4}{3}$

Let $z = t$, where t is any real number. Then $y + \dfrac{3}{2}z = -\dfrac{4}{3} \Rightarrow$

$$y + \frac{3}{2}t = -\frac{4}{3} \Rightarrow y = -\frac{3}{2}t - \frac{4}{3}$$

Row 1 reads $x - 3y - 5z = 7$

Since $y = -\dfrac{3}{2}t - \dfrac{4}{3}$ and $z = t$, then $x - 3\left(-\dfrac{3}{2}t - \dfrac{4}{3}\right) - 5t = 7 \Rightarrow$

$$x + \frac{9}{2}t + 4 - \frac{10}{2}t = 7 \Rightarrow x - \frac{1}{2}t + 4 = 7 \Rightarrow$$

$$x = \frac{1}{2}t + 3$$

Answer: $\left(\frac{1}{2}t + 3,\ -\frac{3}{2}t - \frac{4}{3},\ t\right) = \left(\frac{t+6}{2},\ -\frac{9t+8}{6},\ t\right)$, where t is any real number

1b. $$\begin{aligned} 2x + 5y + 3z &= -8 \\ 4x + 11y + 2z &= -14 \\ -6x - 14y - 13z &= 22 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\left[\begin{array}{ccc|c} 2 & 5 & 3 & -8 \\ 4 & 11 & 2 & -14 \\ -6 & -14 & -13 & 22 \end{array}\right]$$

$$\left[\begin{array}{cccc} 2 & 5 & 3 & -8 \\ 4 & 11 & 2 & -14 \\ -6 & -14 & -13 & 22 \end{array}\right] \xrightarrow[3R_1 + R_3]{-2R_1 + R_2} \left[\begin{array}{cccc} 2 & 5 & 3 & -8 \\ 0 & 1 & -4 & 2 \\ 0 & 1 & -4 & -2 \end{array}\right] \xrightarrow{-R_1 + R_3}$$

$$\left[\begin{array}{cccc} 2 & 5 & 3 & -8 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

Row 3 reads $0x + 0y + 0z = -4$, which is a false equation.

Answer: No Solution

1c. $\begin{aligned} x - 2y + 5z &= 9 \\ -4x + 7y - 23z &= -43 \\ 3x - 5y + 18z &= 34 \end{aligned}$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} 1 & -2 & 5 & 9 \\ -4 & 7 & -23 & -43 \\ 3 & -5 & 18 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 5 & 9 \\ -4 & 7 & -23 & -43 \\ 3 & -5 & 18 & 34 \end{bmatrix} \xrightarrow[-3R_1 + R_3]{4R_1 + R_2} \begin{bmatrix} 1 & -2 & 5 & 9 \\ 0 & -1 & -3 & -7 \\ 0 & 1 & 3 & 7 \end{bmatrix} \xrightarrow{R_2 + R_3}$$

$$\begin{bmatrix} 1 & -2 & 5 & 9 \\ 0 & -1 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & -2 & 5 & 9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 2 reads $y + 3z = 7$

Let $z = t$, where t is any real number. Then $y + 3z = 7 \Rightarrow$

$y + 3t = 7 \Rightarrow y = 7 - 3t$

Row 1 reads $x - 2y + 5z = 9$

Since $y = 7 - 3t$ and $z = t$, then $x - 2(7 - 3t) + 5t = 9 \Rightarrow$

$$x - 14 + 6t + 5t = 9 \Rightarrow x - 14 + 11t = 9 \Rightarrow x = 23 - 11t$$

Answer: $(23 - 11t,\ 7 - 3t,\ t)$, where t is any real number

1d. $\begin{aligned} 4x - 2y + z &= 12 \\ -3x + 4y - 5z &= -13 \\ 9x - 2y - 2z &= 23 \end{aligned}$

Back to Problem 1.

$$\begin{bmatrix} 4 & -2 & 1 & 12 \\ -3 & 4 & -5 & -13 \\ 9 & -2 & -2 & 23 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 2 & -4 & -1 \\ -3 & 4 & -5 & -13 \\ 9 & -2 & -2 & 23 \end{bmatrix} \xrightarrow[-9R_1 + R_3]{3R_1 + R_2}$$

$$\begin{bmatrix} 1 & 2 & -4 & -1 \\ 0 & 10 & -17 & -16 \\ 0 & -20 & 34 & 32 \end{bmatrix} \xrightarrow{2R_2 + R_3} \begin{bmatrix} 1 & 2 & -4 & -1 \\ 0 & 10 & -17 & -16 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3: $0 = 0$ True Equation.

Row 2: $10y - 17z = -16$

Let $z = t$, where t is any real number

$$10y - 17z = -16,\ z = t \Rightarrow 10y - 17t = -16 \Rightarrow 10y = 17t - 16 \Rightarrow$$

$$y = \frac{17t - 16}{10}$$

Row 1: $x + 2y - 4z = -1$

$$x + 2y - 4z = -1,\ y = \frac{17t - 16}{10},\ z = t \Rightarrow$$

$$x + 2\left(\frac{17t - 16}{10}\right) - 4t = -1 \Rightarrow x + \frac{17t - 16}{5} - \frac{20t}{5} = -1 \Rightarrow$$

$$x + \frac{-3t - 16}{5} = -1 \Rightarrow x - \frac{3t + 16}{5} = -1 \Rightarrow x = \frac{3t + 16}{5} - \frac{5}{5} \Rightarrow$$

$$x = \frac{3t + 11}{5}$$

Answer: $\left(\frac{3t + 11}{5}, \frac{17t - 16}{10}, t\right)$, where t is any real number

When $t = 0$, $\left(\frac{3t + 11}{5}, \frac{17t - 16}{10}, t\right) = \left(\frac{11}{5}, -\frac{8}{5}, 0\right)$.

When $t = 1$, $\left(\frac{3t + 11}{5}, \frac{17t - 16}{10}, t\right) = \left(\frac{14}{5}, \frac{1}{10}, 1\right)$.

When $t = \sqrt{2}$, $\left(\frac{3t + 11}{5}, \frac{17t - 16}{10}, t\right) = \left(\frac{3\sqrt{2} + 11}{5}, \frac{17\sqrt{2} - 16}{10}, \sqrt{2}\right)$.

When $t = 5$, $\left(\frac{3t + 11}{5}, \frac{17t - 16}{10}, t\right) = \left(\frac{26}{5}, \frac{69}{10}, 5\right)$.