Solutions for In-Class Problems 18 for Wednesday, April 4

These problems are from Pre-Class Problems 18.

1. Solve the following systems of equations.

a. $\begin{aligned} x^2 + y^2 &= 18 \\ 2x - y &= 3 \end{aligned}$

b. $\begin{aligned} y &= 3x^2 \\ y &= 10x - 8 \end{aligned}$

c. $\begin{aligned} x^2 + y^2 &= 12 \\ (x-3)^2 + y^2 &= 9 \end{aligned}$

d. $\begin{aligned} x &= y^2 - 8 \\ 3x &= -y^2 \end{aligned}$

2. Solve the following systems of equations using Gaussian elimination.

a. $\begin{aligned} 4x - 3y + 5z &= -10 \\ x + 6y - 2z &= 2 \\ -2x - 9y + 3z &= -2 \end{aligned}$

b. $\begin{aligned} 3x - 5y &= 14 \\ 2x + 3z &= 18 \\ -y + 2z &= 9 \end{aligned}$

SOLUTIONS:

1a. $\begin{aligned} x^2 + y^2 &= 18 \\ 2x - y &= 3 \end{aligned}$

Back to Problem 1.

$2x - y = 3 \Rightarrow y = 2x - 3$

$x^2 + y^2 = 18$ and $y = 2x - 3 \Rightarrow x^2 + (2x-3)^2 = 18 \Rightarrow$

$x^2 + 4x^2 - 12x + 9 = 18 \Rightarrow 5x^2 - 12x - 9 = 0 \Rightarrow$

$(x-3)(5x+3)=0 \Rightarrow x=3,\ x=-\frac{3}{5}$

$y=2x-3$ and $x=3 \Rightarrow y=6-3=3$

$y=2x-3$ and $x=-\frac{3}{5} \Rightarrow y=-\frac{6}{5}-\frac{15}{5}=-\frac{21}{5}$

Answer: $(3,\ 3),\left(-\frac{3}{5},-\frac{21}{5}\right)$

1b.

$y=3x^2$
$y=10x-8$

Back to Problem 1.

$y=3x^2$ and $y=10x-8 \Rightarrow 3x^2=10x-8 \Rightarrow$

$3x^2-10x+8=0 \Rightarrow (x-2)(3x-4)=0 \Rightarrow x=2,\ x=\frac{4}{3}$

$y=3x^2$ and $x=2 \Rightarrow y=3(4)=12$

$y=3x^2$ and $x=\frac{4}{3} \Rightarrow y=3\left(\frac{16}{9}\right)=\frac{16}{3}$

Answer: $(2,\ 12),\left(\frac{4}{3},\frac{16}{3}\right)$

1c. $x^2 + y^2 = 12$
$(x - 3)^2 + y^2 = 9$

Back to Problem 1.

$x^2 + y^2 = 12 \Rightarrow y^2 = 12 - x^2$

$(x - 3)^2 + y^2 = 9$ and $y^2 = 12 - x^2 \Rightarrow (x - 3)^2 + 12 - x^2 = 9$

$\Rightarrow x^2 - 6x + 9 + 12 - x^2 = 9 \Rightarrow -6x + 12 = 0 \Rightarrow$

$\Rightarrow 12 = 6x \Rightarrow x = 2$

$x^2 + y^2 = 12$ and $x = 2 \Rightarrow 4 + y^2 = 12 \Rightarrow y^2 = 8 \Rightarrow$

$y = \pm\sqrt{8} = \pm 2\sqrt{2}$

Answer: $(2, 2\sqrt{2}), (2, -2\sqrt{2})$

1d. $x = y^2 - 8$
$3x = -y^2$

Back to Problem 1.

$3x = -y^2 \Rightarrow y^2 = -3x$

$x = y^2 - 8$ and $y^2 = -3x \Rightarrow x = -3x - 8 \Rightarrow 4x = -8 \Rightarrow$

$x = -2$

$y^2 = -3x$ and $x = -2 \Rightarrow y^2 = 6 \Rightarrow y = \pm\sqrt{6}$

Answer: $(-2, \sqrt{6}), (-2, -\sqrt{6})$

2a.

$$\begin{aligned} 4x - 3y + 5z &= -10 \\ x + 6y - 2z &= 2 \\ -2x - 9y + 3z &= -2 \end{aligned}$$

Back to Problem 2.

$$\begin{bmatrix} 4 & -3 & 5 & -10 \\ 1 & 6 & -2 & 2 \\ -2 & -9 & 3 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 6 & -2 & 2 \\ 4 & -3 & 5 & -10 \\ -2 & -9 & 3 & -2 \end{bmatrix} \xrightarrow[2R_1 + R_3]{-4R_1 + R_2} \begin{bmatrix} 1 & 6 & -2 & 2 \\ 0 & -27 & 13 & -18 \\ 0 & 3 & -1 & 2 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 6 & -2 & 2 \\ 0 & 3 & -1 & 2 \\ 0 & -27 & 13 & -18 \end{bmatrix} \xrightarrow{9R_2 + R_3} \begin{bmatrix} 1 & 6 & -2 & 2 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

Row 3: $4z = 0 \Rightarrow z = 0$

Row 2: $3y - z = 2$

$3y - z = 2, z = 0 \Rightarrow 3y = 2 \Rightarrow y = \frac{2}{3}$

Row 1: $x + 6y - 2z = 2$

$$x + 6y - 2z = 2,\ y = \frac{2}{3},\ z = 0 \Rightarrow x + 4 - 0 = 2 \Rightarrow x = -2$$

Answer: $\left(-2, \frac{2}{3}, 0\right)$

2b.
$$\begin{aligned} 3x - 5y &= 14 \\ 2x + 3z &= 18 \\ -y + 2z &= 9 \end{aligned}$$

Back to Problem 2.

$$\begin{bmatrix} 3 & -5 & 0 & 14 \\ 2 & 0 & 3 & 18 \\ 0 & -1 & 2 & 9 \end{bmatrix} \xrightarrow{-R_2 + R_1} \begin{bmatrix} 1 & -5 & -3 & -4 \\ 2 & 0 & 3 & 18 \\ 0 & -1 & 2 & 9 \end{bmatrix} \xrightarrow[-R_3]{-2R_1 + R_2} \begin{bmatrix} 1 & -5 & -3 & -4 \\ 0 & 10 & 9 & 26 \\ 0 & 1 & -2 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -5 & -3 & -4 \\ 0 & 1 & -2 & -9 \\ 0 & 10 & 9 & 26 \end{bmatrix} \xrightarrow{-10R_2 + R_3} \begin{bmatrix} 1 & -5 & -3 & -4 \\ 0 & 1 & -2 & -9 \\ 0 & 0 & 29 & 116 \end{bmatrix} \xrightarrow{\frac{1}{29}R_3} \begin{bmatrix} 1 & -5 & -3 & -4 \\ 0 & 1 & -2 & -9 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Row 3: $z = 4$

Row 2: $y - 2z = -9$

$y - 2z = -9,\ z = 4 \Rightarrow y - 8 = -9 \Rightarrow y = -1$

Row 1: $x - 5y - 3z = -4$

$x - 5y - 3z = -4,\ y = -1,\ z = 4 \Rightarrow x + 5 - 12 = -4 \Rightarrow$

$x - 7 = -4 \Rightarrow x = 3$

Answer: $(3,\ -1,\ 4)$