Solutions for In-Class Problems 17 for Monday, April 2

These problems are from Pre-Class Problems 17.

1. Solve the following exponential equations.

   a. $27^{x-5} = \dfrac{1}{81}$

   b. $2^{4x+5} = 7$

   c. $4^{6x-7} = 12^{8-x}$

2. Solve the following system of equations by using the substitution method.

   a. $\begin{array}{l} 3x - 8y = 22 \\ x - 16y = 38 \end{array}$

   b. $\begin{array}{l} 4x + y = -7 \\ 3x + 5y = 16 \end{array}$

3. Solve the following system of equations by using the addition method.

   a. $\begin{array}{l} 4x + 3y = 14 \\ -5x + 6y = -24 \end{array}$

   b. $\begin{array}{l} 3x - 5y = 3 \\ 7x + 3y = -4 \end{array}$

4. Solve the following systems of equations.

   a. $\begin{array}{l} 4x - 3y + 5z = -10 \\ x + 6y - 2z = 2 \\ -2x - 9y + 3z = -2 \end{array}$

   b. $\begin{array}{l} 3x - 5y = 14 \\ 2x + 3z = 18 \\ -y + 2z = 9 \end{array}$

   c. $\begin{array}{l} 4x - 2y + z = 12 \\ -3x + 4y - 5z = -13 \\ 9x - 2y - 2z = 23 \end{array}$

5. A citrus fruit grower purchased 30 orange trees and 20 lemon trees for $360. The next week the grower bought 25 orange trees and 35 lemon trees for $410. Find the cost of a lemon tree.

SOLUTIONS:

1a.　$27^{x-5} = \dfrac{1}{81}$　　　　　　　　　　　Back to .

$27^{x-5} = \dfrac{1}{81} \Rightarrow (3^3)^{x-5} = 3^{-4} \Rightarrow 3^{3(x-5)} = 3^{-4} \Rightarrow$

$3(x-5) = -4 \Rightarrow 3x - 15 = -4 \Rightarrow x = \dfrac{11}{3}$

Answer: $x = \dfrac{11}{3}$

1b.　$2^{4x+5} = 7$　　　　　　　　　　　Back to .

$2^{4x+5} = 7 \Rightarrow \ln 2^{4x+5} = \ln 7 \Rightarrow (4x+5)\ln 2 = \ln 7 \Rightarrow$

$4x\ln 2 + 5\ln 2 = \ln 7 \Rightarrow 4x\ln 2 = \ln 7 - 5\ln 2 \Rightarrow$

$x = \dfrac{\ln 7 - 5\ln 2}{4\ln 2} = \dfrac{\ln 7 - \ln 32}{\ln 16} = \dfrac{\ln \dfrac{7}{32}}{\ln 16}$

Answer: $x = \dfrac{\ln \dfrac{7}{32}}{\ln 16}$

1c.　$4^{6x-7} = 12^{8-x}$　　　　　　　　　　　Back to .

$$4^{6x-7} = 12^{8-x} \Rightarrow \ln 4^{6x-7} = \ln 12^{8-x} \Rightarrow (6x-7)\ln 4 = (8-x)\ln 12 \Rightarrow$$

$$6x\ln 4 - 7\ln 4 = 8\ln 12 - x\ln 12 \Rightarrow 6x\ln 4 + x\ln 12 = 8\ln 12 + 7\ln 4 \Rightarrow$$

$$x(6\ln 4 + \ln 12) = 8\ln 12 + 7\ln 4 \Rightarrow x = \frac{8\ln 12 + 7\ln 4}{6\ln 4 + \ln 12}$$

Answer: $x = \dfrac{8\ln 12 + 7\ln 4}{6\ln 4 + \ln 12}$

2a. $\begin{array}{l} 3x - 8y = 22 \\ x - 16y = 38 \end{array}$

$$x - 16y = 38 \Rightarrow x = 16y + 38$$

$$3x - 8y = 22 \text{ and } x = 16y + 38 \Rightarrow 3(16y + 38) - 8y = 22 \Rightarrow$$

$$48y + 114 - 8y = 22 \Rightarrow 40y = -92 \Rightarrow y = -\frac{92}{40} = -\frac{46}{20} = -\frac{23}{10}$$

$$x = 16y + 38 \text{ and } y = -\frac{23}{10} \Rightarrow x = 16\left(-\frac{23}{10}\right) + 38 = 8\left(-\frac{23}{5}\right) + 38 =$$

$$-\frac{184}{5} + \frac{190}{5} = \frac{6}{5}$$

Answer: $\left(\dfrac{6}{5}, -\dfrac{23}{10} \right)$

2b.
$$4x + y = -7$$
$$3x + 5y = 16$$

$$4x + y = -7 \implies y = -4x - 7$$

$$3x + 5y = 16 \text{ and } y = -4x - 7 \implies 3x + 5(-4x - 7) = 16 \implies$$

$$3x - 20x - 35 = 16 \implies -17x - 35 = 16 \implies -17x = 51 \implies x = -3$$

$$y = -4x - 7 \text{ and } x = -3 \implies y = 12 - 7 = 5$$

Answer: $(-3, 5)$

3a.
$$4x + 3y = 14$$
$$-5x + 6y = -24$$

$$\begin{array}{l} 4x + 3y = 14 \\ -5x + 6y = -24 \end{array} \implies \begin{array}{l} -8x - 6y = -28 \\ -5x + 6y = -24 \\ \hline -13x \qquad = -52 \implies x = 4 \end{array}$$

$$4x + 3y = 14 \text{ and } x = 4 \implies 16 + 3y = 14 \implies 3y = -2 \implies$$
$$y = -\frac{2}{3}$$

Answer: $\left(4, -\frac{2}{3} \right)$

3b.
$$3x - 5y = 3$$
$$7x + 3y = -4$$

$$\begin{aligned} 3x - 5y &= 3 \\ 7x + 3y &= -4 \end{aligned} \Rightarrow \begin{aligned} 9x - 15y &= 9 \\ 35x + 15y &= -20 \\ \hline 44x \qquad\quad &= -11 \end{aligned} \Rightarrow x = -\frac{11}{44} = -\frac{1}{4}$$

$$3x - 5y = 3 \text{ and } x = -\frac{1}{4} \Rightarrow -\frac{3}{4} - 5y = \frac{12}{4} \Rightarrow -5y = \frac{15}{4} \Rightarrow$$

$$y = -\frac{3}{4}$$

Answer: $\left(-\dfrac{1}{4}, \ -\dfrac{3}{4} \right)$

4a.
$$\begin{aligned} 4x - 3y + 5z &= -10 \\ x + 6y - 2z &= 2 \\ -2x - 9y + 3z &= -2 \end{aligned}$$

$$\begin{aligned} 4x - 3y + 5z &= -10 \\ x + 6y - 2z &= 2 \end{aligned} \Rightarrow \begin{aligned} 4x - 3y + 5z &= -10 \\ -4x - 24y + 8z &= -8 \\ \hline -27y + 13z &= -18 \end{aligned}$$

$$\begin{aligned} x + 6y - 2z &= 2 \\ -2x - 9y + 3z &= -2 \end{aligned} \Rightarrow \begin{aligned} 2x + 12y - 4z &= 4 \\ -2x - 9y + 3z &= -2 \\ \hline 3y - \quad z &= 2 \end{aligned}$$

$$\begin{aligned} -27y + 13z &= -18 \\ 3y - \quad z &= 2 \end{aligned} \quad \Rightarrow \quad \begin{array}{l} -27y + 13z = -18 \\ \underline{39y - 13z = 26} \\ 12y = 8 \end{array}$$

$$12y = 8 \;\Rightarrow\; y = \frac{8}{12} = \frac{2}{3}$$

$$3y - z = 2 \text{ and } y = \frac{2}{3} \;\Rightarrow\; 2 - z = 2 \;\Rightarrow\; z = 0$$

$$x + 6y - 2z = 2 \text{ and } y = \frac{2}{3} \text{ and } z = 0 \;\Rightarrow$$

$$x + 4 - 0 = 2 \;\Rightarrow\; x + 4 = 2 \;\Rightarrow\; x = -2$$

Answer: $\left(-2,\, \dfrac{2}{3},\, 0 \right)$

4b.
$$\begin{aligned} 3x - 5y &= 14 \\ 2x + 3z &= 18 \\ -y + 2z &= 9 \end{aligned}$$

$$\begin{aligned} 3x - 5y &= 14 \\ - \quad y + 2z &= 9 \end{aligned} \quad \Rightarrow \quad \begin{array}{l} 3x - 5y = 14 \\ \underline{5y - 10z = -45} \\ 3x - 10z = -31 \end{array}$$

$$6x + 9z = 54$$

$$
\begin{aligned}
2x + 3z &= 18 \\
3x - 10z &= -31
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
6x + 9z &= 54 \\
-6x + 20z &= 62 \\
\hline
29z &= 116 \Rightarrow z = 4
\end{aligned}
$$

$2x + 3z = 18$ and $z = 4 \Rightarrow 2x + 12 = 18 \Rightarrow 2x = 6 \Rightarrow x = 3$

$-y + 2z = 9$ and $z = 4 \Rightarrow -y + 8 = 9 \Rightarrow y = -1$

Answer: $(3, -1, 4)$

4c.
$$
\begin{aligned}
4x - 2y + z &= 12 \\
-3x + 4y - 5z &= -13 \\
9x - 2y - 2z &= 23
\end{aligned}
$$

$$
\begin{aligned}
4x - 2y + z &= 12 \\
-3x + 4y - 5z &= -13
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
20x - 10y + 5z &= 60 \\
-3x + 4y - 5z &= -13 \\
\hline
17x - 6y &= 47
\end{aligned}
$$

$$
\begin{aligned}
4x - 2y + z &= 12 \\
9x - 2y - 2z &= 23
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
8x - 4y + 2z &= 24 \\
9x - 2y - 2z &= 23 \\
\hline
17x - 6y &= 47
\end{aligned}
$$

$17x - 6y = 47$

$17x - 6y = 47$

These two equations are the same equation.

Let $y = t$, where t is any real number.

$17x - 6y = 47$ and $y = t \Rightarrow 17x - 6t = 47 \Rightarrow 17x = 6t + 47 \Rightarrow$

$$x = \frac{6t + 47}{17} = \frac{6}{17}t + \frac{47}{17}$$

Now, use the first equation in the original system of equations to find the value of z when $x = \frac{6}{17}t + \frac{47}{17}$ and $y = t$:

$4x - 2y + z = 12$, $x = \frac{6}{17}t + \frac{47}{17}$, and $y = t \Rightarrow$

$$\frac{24}{17}t + \frac{188}{17} - 2t + z = 12 \Rightarrow \frac{24}{17}t + \frac{188}{17} - \frac{34}{17}t + z = 12 \Rightarrow$$

$$-\frac{10}{17}t + \frac{188}{17} + z = \frac{204}{17} \Rightarrow z = \frac{10}{17}t + \frac{16}{17}$$

Answer: $\left(\frac{6}{17}t + \frac{47}{17},\ t,\ \frac{10}{17}t + \frac{16}{17} \right) = \left(\frac{6t + 47}{17},\ t,\ \frac{10t + 16}{17} \right)$, where t is any real number

5. A citrus fruit grower purchased 20 orange trees and 30 lemon trees for $360. The next week the grower bought 35 orange trees and 25 lemon trees for $410. Find the cost of a lemon tree.

Let o = the cost of one orange tree and let l = the cost of one lemon tree.

$$\begin{aligned} 20o + 30l &= 360 \\ 35o + 25l &= 410 \end{aligned} \Rightarrow \begin{aligned} 2o + 3l &= 36 \\ 7o + 5l &= 82 \end{aligned} \Rightarrow \begin{aligned} 14o + 21l &= 252 \\ \underline{-14o - 10l} &= \underline{-164} \\ 11l &= 88 \Rightarrow l = 8 \end{aligned}$$

Answer: $8.00