Solutions for In-Class Problems 16 for Wednesday, March 28

These problems are from Pre-Class Problems 16.

1. Use the properties of logarithms to write the following as a sum and/or difference of logarithms. All variables represent positive numbers.

a. $\ln \dfrac{x^4}{\sqrt[3]{y^2}}$

b. $\log \left[(4y^3 + 5y^2 - 8)^7 \sqrt{4y^2 - 9} \right]$

c. $\log_5 \dfrac{6 - w^2}{3w + 2}$

d. $\log_{2/3} \dfrac{x^2 \sqrt[5]{2x - 7}}{(x - 8)^3 (5x^4 + 11)}$

2. Write the following as a single logarithm.

a. $\ln x + 5\ln (x^2 - 16) - \dfrac{3}{2}\ln (9x + 8)$

b. $2\log_{1/2} y - \log_{1/2}(3y - 5) - \dfrac{1}{4}\log_{1/2}(y^3 - 27) + \log_{1/2}(y + 4)$

3. Use the change of base formula and a calculator to approximate the following to four decimal places (the nearest ten-thousandth).

a. $\log_3 85$

b. $\log_5 \dfrac{3}{7}$

c. $\log_{1/2} 9$

4. Solve the following logarithmic equations.

a. $\log (3x + 7) = 1$

b. $\log_6 x = 2 - \log_6 (x - 9)$

c. $\ln (40 - t) = \ln (5t + 12)$

SOLUTIONS:

1a. $\ln \dfrac{x^4}{\sqrt[3]{y^2}}$ Back to Problem 1.

$$\ln \frac{x^4}{\sqrt[3]{y^2}} = \ln x^4 - \ln y^{2/3} = 4\ln x - \frac{2}{3}\ln y$$

Answer: $4\ln x - \dfrac{2}{3}\ln y$

1b. $\log [\,(4y^3 + 5y^2 - 8)^7 \sqrt{4y^2 - 9}\,]$ Back to Problem 1.

$$\log [\,(4y^3 + 5y^2 - 8)^7 \sqrt{4y^2 - 9}\,] =$$

$$\log (4y^3 + 5y^2 - 8)^7 + \log (4y^2 - 9)^{1/2} =$$

$$7 \log (4y^3 + 5y^2 - 8)^7 + \frac{1}{2} \log (4y^2 - 9)$$

Answer: $7 \log (4y^3 + 5y^2 - 8)^7 + \dfrac{1}{2} \log (4y^2 - 9)$

1c. $\log_5 \dfrac{6 - w^2}{3w + 2}$ Back to Problem 1.

$$\log_5 \frac{6 - w^2}{3w + 2} = \log_5 (6 - w^2) - \log_5 (3w + 2)$$

Answer: $\log_5 (6 - w^2) - \log_5 (3w + 2)$

1d. $\log_{2/3} \frac{x^2 \sqrt[5]{2x - 7}}{(x - 8)^3 (5x^4 + 11)}$

Back to Problem 1.

$$\log_{2/3} \frac{x^2 \sqrt[5]{2x - 7}}{(x - 8)^3 (5x^4 + 11)} =$$

$$\log_{2/3} x^2 + \log_{2/3} (2x - 7)^{1/5} - \log_{2/3} (x - 8)^3 - \log_{2/3} (5x^4 + 11) =$$

$$2 \log_{2/3} x + \frac{1}{5} \log_{2/3} (2x - 7) - 3 \log_{2/3} (x - 8) - \log_{2/3} (5x^4 + 11)$$

Answer: $2 \log_{2/3} x + \frac{1}{5} \log_{2/3} (2x - 7) - 3 \log_{2/3} (x - 8) - \log_{2/3} (5x^4 + 11)$

2a. $\ln x + 5 \ln (x^2 - 16) - \frac{3}{2} \ln (9x + 8)$

$$\ln x + 5 \ln (x^2 - 16) - \frac{3}{2} \ln (9x + 8) =$$

$$\ln x + \ln (x^2 - 16)^5 - \ln (9x + 8)^{3/2} = \ln \frac{x (x^2 - 16)^5}{(9x + 8)^{3/2}} =$$

$$\ln \frac{x(x^2 - 16)^5}{\sqrt{(9x + 8)^3}}$$

Answer: $\ln \frac{x(x^2 - 16)^5}{\sqrt{(9x + 8)^3}}$ Back to Problem 2.

2b. $2\log_{1/2} y - \log_{1/2}(3y - 5) - \frac{1}{4}\log_{1/2}(y^3 - 27) + \log_{1/2}(y + 4)$

$$2\log_{1/2} y - \log_{1/2}(3y - 5) - \frac{1}{4}\log_{1/2}(y^3 - 27) + \log_{1/2}(y + 4) =$$

$$\log_{1/2} y^2 - \log_{1/2}(3y - 5) - \log_{1/2}(y^3 - 27)^{1/4} + \log_{1/2}(y + 4) =$$

$$\log_{1/2} \frac{y^2(y + 4)}{(3y - 5)(y^3 - 27)^{1/4}} = \log_{1/2} \frac{y^2(y + 4)}{(3y - 5)\sqrt[4]{y^3 - 27}}$$

Answer: $\log_{1/2} \frac{y^2(y + 4)}{(3y - 5)\sqrt[4]{y^3 - 27}}$ Back to Problem 2.

3a. $\log_3 85$ Back to Problem 3.

$$\log_3 85 = \frac{\ln 85}{\ln 3} \approx 4.0439$$

Answer: 4.4039

3b. $\log_5 \frac{3}{7}$ Back to Problem 3.

$$\log_5 \frac{3}{7} = \frac{\ln (3/7)}{\ln 5} \approx -0.5265$$

Answer: -0.5265

3c. $\log_{1/2} 9$ Back to Problem 3.

$$\log_{1/2} 9 = \frac{\ln 9}{\ln (1/2)} \approx -3.1699$$

NOTE: Since $\ln (1/2) = \ln 1 - \ln 2 = 0 - \ln 2 = -\ln 2$ or

$\ln (1/2) = \ln 2^{-1} = -\ln 2$, then

$$\log_{1/2} 9 = \frac{\ln 9}{\ln (1/2)} = \frac{\ln 9}{-\ln 2} = -\frac{\ln 9}{\ln 2}$$

Answer: -3.1699

4a. $\log (3x + 7) = 1$ Back to Problem 4.

$\log(3x+7)=1 \Rightarrow 3x+7=10^1 \Rightarrow 3x+7=10 \Rightarrow 3x=3 \Rightarrow$

$x=1$

Answer: $x=1$

4b. $\log_6 x = 2 - \log_6(x-9)$ Back to Problem 4.

$\log_6 x = 2 - \log_6(x-9) \Rightarrow \log_6 x + \log_6(x-9) = 2 \Rightarrow$

$\log_6 x(x-9) = 2 \Rightarrow x(x-9) = 6^2 \Rightarrow x^2 - 9x = 36 \Rightarrow$

$x^2 - 9x - 36 = 0 \Rightarrow (x+3)(x-12) = 0 \Rightarrow x = -3,\ x = 12$

When $x=-3$, $\log_6 x = \log_6(-3)$, which is undefined.

When $x=12$, $x = 12 > 0$ and $x - 9 = 3 > 0$.

Answer: $x=12$

4c. $\ln(40-t) = \ln(5t+12)$ Back to Problem 4.

$\ln(40-t) = \ln(5t+12) \Rightarrow 40 - t = 5t + 12 \Rightarrow 28 = 6t \Rightarrow$

$t = \dfrac{14}{3}$

When $t = \frac{14}{3}$, $40 - t > 0$ and $5t + 12 > 0$.

Answer: $t = \frac{14}{3}$