Solutions for In-Class Problems 14 for Wednesday, March 21

These problems are from Pre-Class Problems 14.

1. Identity the horizontal asymptotes (if any). If the function has a horizontal asymptote, determine if the graph crosses the asymptote.

a. $h(x) = \dfrac{x^2 + 5x}{4x^2 + 9x - 12}$ b. $f(x) = \dfrac{x^3 - 6x^2 + 9}{2x^2 + 7x - 14}$

2. Determine the vertical and horizontal asymptotes for the graph of the following rational functions (if any). If the function has a horizontal asymptote, determine if the graph crosses the asymptote. Then sketch the graph of the rational function.

a. $f(x) = \dfrac{2x + 5}{3x - 7}$ b. $g(x) = \dfrac{8x}{x^2 - 16}$

3. Sketch the graph of the following functions. State the domain of the function and use the sketch to state the range of the function.

a. $f(x) = 5\left(\dfrac{2}{3}\right)^{x - 4}$ b. $y = 7^{t + 4} + 2$

c. $g(x) = -4e^{-x} - 9$

SOLUTIONS:

1a. $h(x) = \dfrac{x^2 + 5x}{4x^2 + 9x - 12}$

Back to Problem 1.

$$h(x) = \frac{x^2\left(1 + \frac{5}{x}\right)}{x^2\left(4 + \frac{9}{x} - \frac{12}{x^2}\right)} = \frac{1 + \frac{5}{x}}{4 + \frac{9}{x} - \frac{12}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $\frac{5}{x} \to 0$, $\frac{9}{x} \to 0$, and $\frac{12}{x^2} \to 0$. Thus, as $x \to -\infty$ and as $x \to \infty$, $h(x) \to \frac{1 + 0}{4 + 0 - 0} = \frac{1}{4}$.

Thus, $y = \frac{1}{4}$ is a horizontal asymptotes for the graph of the rational function *h*.

The graph of the rational function *h* will cross the horizontal asymptote $y = \frac{1}{4}$ if the equation $h(x) = \frac{1}{4}$ has a real number solution.

$$h(x) = \frac{1}{4} \Rightarrow \frac{x^2 + 5x}{4x^2 + 9x - 12} = \frac{1}{4} \Rightarrow 4x^2 + 9x - 12 =$$

$$4(x^2 + 5x) \Rightarrow 4x^2 + 9x - 12 = 4x^2 + 20x \Rightarrow$$

$$9x - 12 = 20x \Rightarrow -12 = 11x \Rightarrow x = -\frac{12}{11}.$$

The graph of the rational function *h* will cross the horizontal asymptote $y = \frac{1}{4}$ at the point $\left(-\frac{12}{11}, \frac{1}{4}\right)$.

Answer: Horizontal Asymptote: $y = \frac{1}{4}$

Graph crosses horizontal asymptote at $\left(-\frac{12}{11}, \frac{1}{4} \right)$

1b. $f(x) = \frac{x^3 - 6x^2 + 9}{2x^2 + 7x - 14}$ Back to Problem 1.

$$f(x) = \frac{2x^2 + 7x - 14}{x^3 - 6x^2 + 9}$$

$$f(x) = \frac{x^3\left(\frac{2}{x} + \frac{7}{x^2} - \frac{14}{x^3}\right)}{x^3\left(1 - \frac{6}{x} + \frac{9}{x^3}\right)} = \frac{\frac{2}{x} + \frac{7}{x^2} - \frac{14}{x^3}}{1 - \frac{6}{x} + \frac{9}{x^3}}$$

As $x \to -\infty$ and as $x \to \infty$, $f(x) \to \frac{0 + 0 - 0}{1 - 0 + 0} = \frac{0}{1} = 0$.

Thus, $y = 0$ is a horizontal asymptotes for the graph of the rational function *f*.

The graph of the rational function *f* will cross the horizontal asymptote $y = 0$ if the equation $f(x) = 0$ has a real number solution.

$$f(x) = 0 \Rightarrow \frac{2x^2 + 7x - 14}{x^3 - 6x^2 + 9} = 0 \Rightarrow 2x^2 + 7x - 14 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{49 - 4(2)(-14)}}{4} =$$

$$\frac{-7 \pm \sqrt{49 + 112}}{4} = \frac{-7 \pm \sqrt{161}}{4}$$

Answer: Horizontal Asymptote: $y = 0$

Graph crosses horizontal asymptote at $\left(\frac{-7 - \sqrt{161}}{4}, 0 \right)$ and $\left(\frac{-7 + \sqrt{161}}{4}, 0 \right)$

2a. I will owe you the solution for this problem. Back to Problem 2.

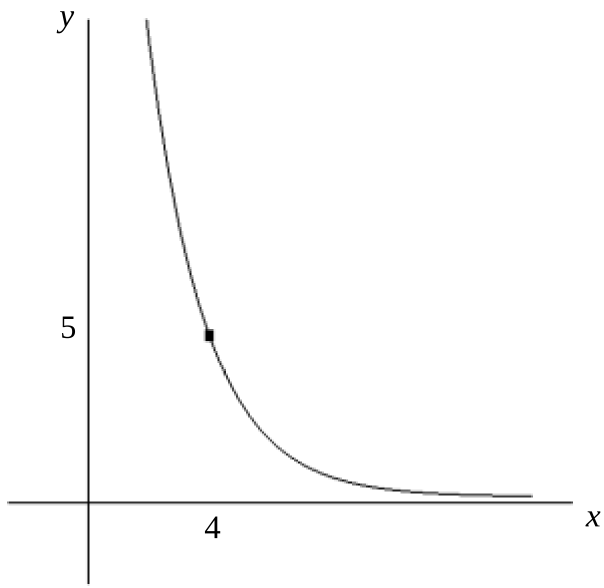
2b. I will owe you the solution for this problem. Back to Problem 2.

3a. $f(x) = 5\left(\frac{2}{3}\right)^{x-4}$ Back to Problem 3.

The domain of f is the set of all real numbers.

To graph the function f, we set $f(x) = y$ and graph the equation $y = 5\left(\frac{2}{3}\right)^{x-4}$.

The graph of $y = 5\left(\frac{2}{3}\right)^{x-4}$ is the graph of $y = 5\left(\frac{2}{3}\right)^{x}$ shifted 4 units to the right.



The range of f is $(0, \infty)$.

NOTE: The *y*-coordinate of the *y*-intercept is obtained by setting $x = 0$ in the equation $y = 5\left(\frac{2}{3}\right)^{x - 4}$. Thus, we have that $y = 5\left(\frac{2}{3}\right)^{-4} = 5\left(\frac{3}{2}\right)^{4}$ $= 5\left(\frac{81}{16}\right) = \frac{405}{16} \approx 25$. Thus, the *y*-intercept is the point $\left(0, \frac{405}{16}\right)$.

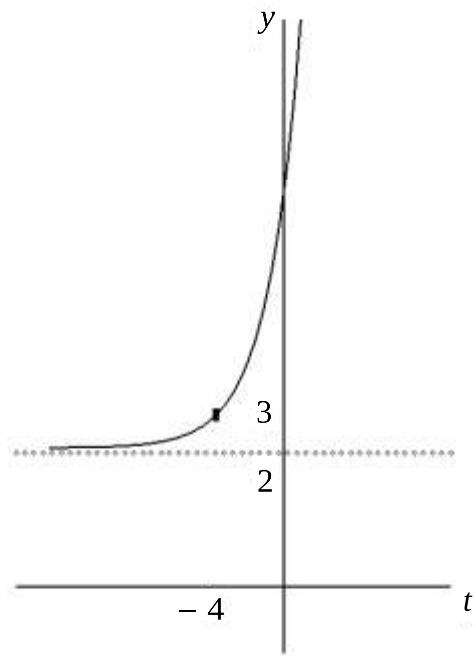
NOTE: The horizontal shift of 4 units to the right is determined from the expression $x - 4$ in the equation $y = 5\left(\frac{2}{3}\right)^{x - 4}$.

3b. $y = 7^{t + 4} + 2$ Back to Problem 3.

The domain of the function is the set of all real numbers.

$$y = 7^{t + 4} + 2 \Rightarrow y - 2 = 7^{t + 4}$$

The graph of $y - 2 = 7^{t+4}$ is the graph of $y = 7^t$ shifted 4 units to the left and 2 units upward.



The range of the function is $(2, \infty)$.

The *y*-coordinate of the *y*-intercept is obtained by setting $4 = 0$ in the equation $y = 7^{t+4} + 2$. Thus, we have that $y = 7^4 + 2 = 2401 + 2 =$ 2403. Thus, the *y*-intercept is the point $(0, 2403)$.

NOTE: The horizontal shift of 4 units to the left is determined from the expression $t + 4$ in the equation $y - 2 = 7^{t+4}$ and the vertical shift of 2 units upward is determined from the expression $y - 2$ in the equation.

3c. $g(x) = -4e^{-x} - 9$ Back to Problem 3.

The domain of g is the set of all real numbers.

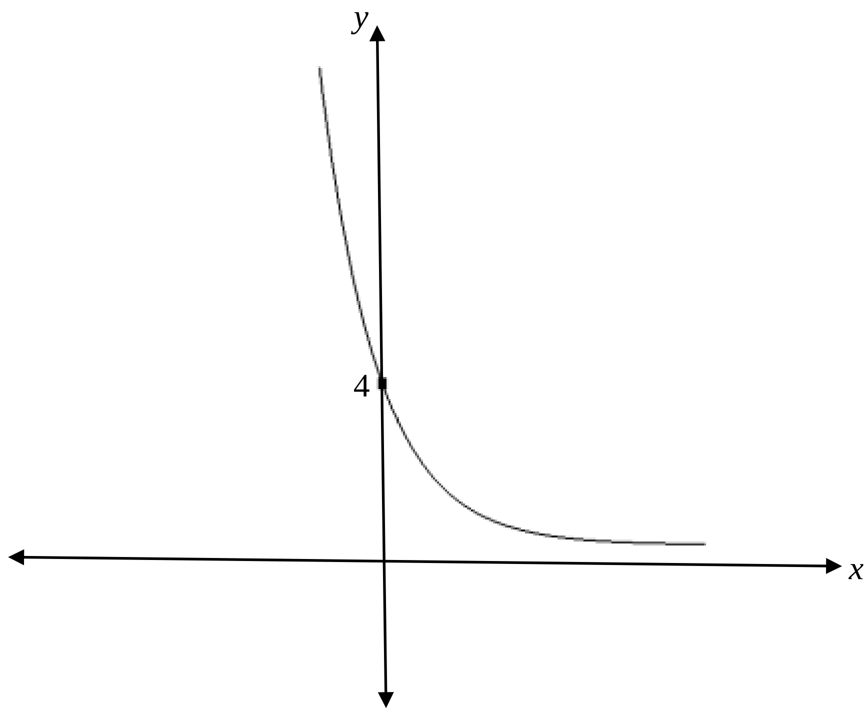
To graph the function g, we set $g(x) = y$ and graph the equation

$y = -4e^{-x} - 9$.

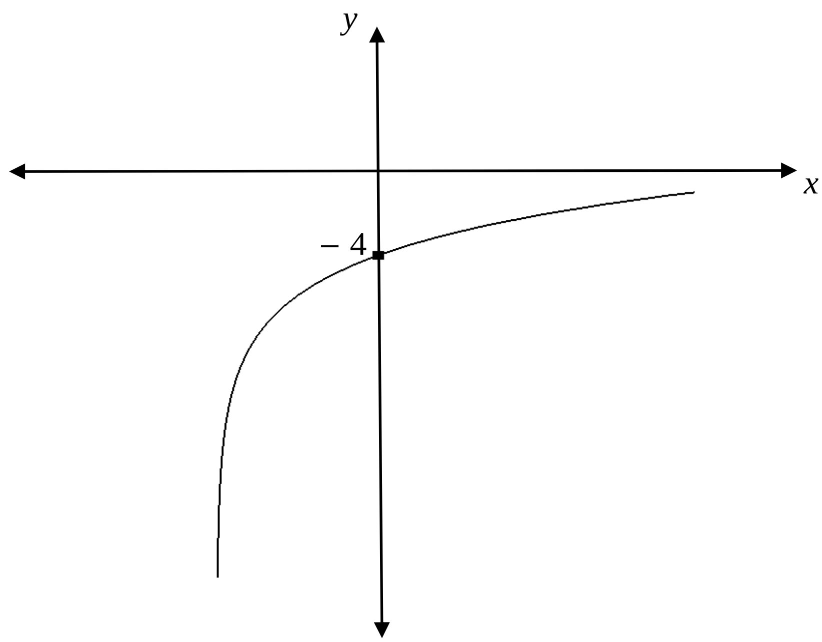
$$y = -4e^{-x} - 9 \Rightarrow y + 9 = -4e^{-x}$$

The graph of $y + 9 = -4e^{-x}$ is the graph of $y = -4e^{-x} = -4\left(\frac{1}{e}\right)^x$ shifted 9 units downward. Since $e \approx 2.7 > 1$, then $\frac{1}{e} < 1$.

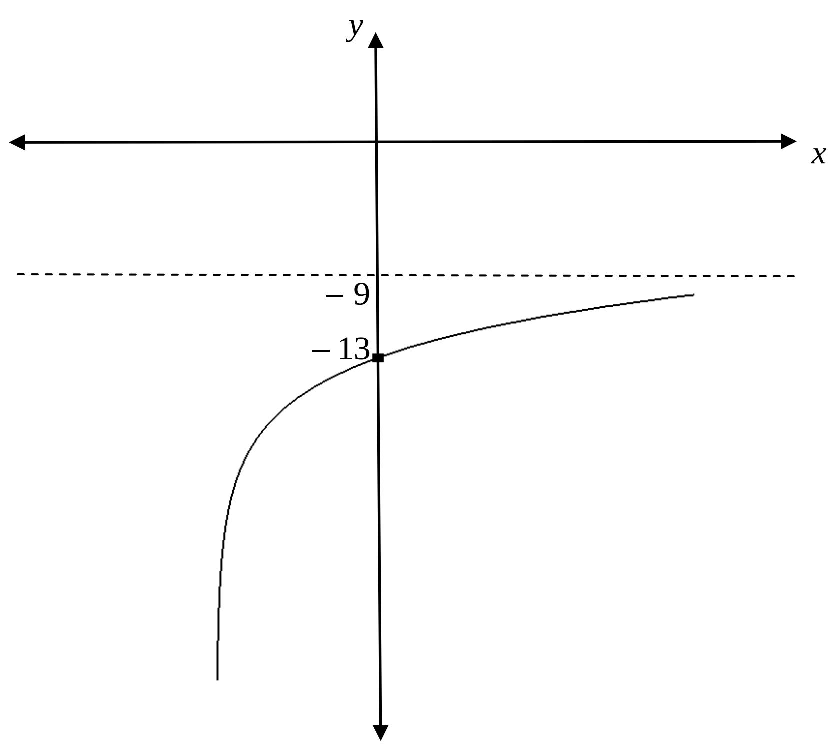
The graph of $y = 4e^{-x} = 4\left(\frac{1}{e}\right)^x$ looks like the following.



The graph of $y = -4e^{-x} = -4\left(\frac{1}{e}\right)^x$ looks like the following.



Now, the graph of $y + 9 = -4e^{-x}$ looks like the following.



The range of g is $(-\infty, -9)$. Note that the *y*-intercept is the point $(0, -13)$.

NOTE: The vertical shift of 9 units downward is determined from the expression $y + 9$ in the equation $y + 9 = -4e^{-x}$.