Solutions for In-Class Problems 12 for Monday, March 12

These problems are from Pre-Class Problems 12.

You can go to the solution for each problem by clicking on the problem letter or problem number.

1. Use synthetic division to divide the following polynomials.

   a. $(2x^3 - 7x^2 + 15x - 20) \div (x + 3)$ b. $\dfrac{x^3 - 125}{x - 5}$

2. If $f(x) = x^4 + 6x^2 - 9x + 12$, then use the Remainder Theorem to find the following polynomial values.

   a. $f(-5)$ b. $f(8)$ c. $f(4i)$

3. If $g(x) = 6x^4 - 11x^3 - 53x^2 + 108x - 36$, then use the Remainder Theorem to determine if the given number is a zero (root) of the polynomial.

   a. -3 b. 3 c. $\dfrac{3}{2}$

4. If $h(x) = 3x^4 + 8x^3 + 44x - 80$, then use the Factor Theorem to determine if the given binomial is a factor of the polynomial.

   a. $x - 2$ b. $x + 4$ c. $x - \dfrac{1}{3}$

5. Find the zeros (roots) of the following polynomials. Also, give a factorization for the polynomial.

   a. $f(x) = 3x^4 - 7x^3 - 20x^2 + 30x + 36$

   b. $g(x) = 2x^4 - 8x^3 + 15x^2 - 28x + 28$

SOLUTIONS:

1a. $(2x^3 - 7x^2 + 15x - 20) \div (x + 3)$

Back to Problem 1.

$$\begin{array}{rrrr|l} 2 & -7 & 15 & -20 & -3 \\ & -6 & 39 & -162 & \\ \hline 2 & -13 & 54 & -182 & \end{array}$$

Answer: $\dfrac{2x^3 - 7x^2 + 15x - 20}{x + 3} = 2x^2 - 13x + 54 - \dfrac{182}{x + 3}$

1b. $\dfrac{x^3 - 125}{x - 5}$

Back to Problem 1.

$$\begin{array}{rrrr|l} 1 & 0 & 0 & -125 & 5 \\ & 5 & 25 & 125 & \\ \hline 1 & 5 & 25 & 0 & \end{array}$$

Answer: $\dfrac{x^3 - 125}{x - 5} = x^2 + 5x + 25$

2a. $f(-5)$

Back to Problem 2.

$$\begin{array}{rrrrr|l} 1 & 0 & 6 & -9 & 12 & -5 \\ & -5 & 25 & -155 & 820 & \\ \hline 1 & -5 & 31 & -164 & 832 & \end{array}$$

Answer: 832

2b. $f(8)$ Back to Problem 2.

1	0	6	-9	12	$\lvert\, 8$
	8	64	560	4408	
1	8	70	551	4420	

Answer: 4420

2c. $f(4i)$ Back to Problem 2.

1	0	6	-9	12	$\lvert\, 4i$
	$4i$	-16	$-40i$	$-36i+160$	
1	$4i$	-10	$-9-40i$	$172-36i$	

Answer: $172 - 36i$

3a.

6	-11	-53	108	-36	$\lvert\, -3$
	-18	87	-102	-18	
6	-29	34	6	-54	

$g(-3) = -54$

Answer: -3 is not a zero (root) of the polynomial

Back to Problem 3.

3b.

6	– 11	– 53	108	– 36	3
	18	21	– 96	36	
6	7	– 32	12	0	

$g(3) = 0$

Answer: 3 is a zero (root) of the polynomial

Back to Problem 3.

3c.

6	– 11	– 53	108	– 36	$\frac{3}{2}$
	9	– 3	– 84	36	
6	– 2	– 56	24	0	

$g\left(\frac{3}{2}\right) = 0$

Answer: $\frac{3}{2}$ is a zero (root) of the polynomial Back to Problem 3.

4a. $x - 2$ Back to Problem 4.

$x - 2$ is a factor of the polynomial h if and only if $h(2) = 0$.

3	8	0	44	– 80	2
	6	28	56	200	
3	14	28	100	120	

$h(2) = 120$

Answer: $x - 2$ is not a factor of the polynomial

4b. $x + 4$ Back to Problem 4.

$x + 4$ is a factor of the polynomial h if and only if $h(-4) = 0$.

3	8	0	44	– 80	– 4
	– 12	16	– 64	80	
3	– 4	16	– 20	0	

$h(-4) = 0$

Answer: $x + 4$ is a factor of the polynomial

4c. $x - \frac{1}{3}$ Back to Problem 4.

$x - \frac{1}{3}$ is a factor of the polynomial h if and only if $h\left(\frac{1}{3}\right) = 0$.

3	8	0	44	– 80	$\frac{1}{3}$
	1	3	1	15	
3	9	3	45	– 65	

$$h\left(\frac{1}{3}\right) = -65$$

Answer: $x - \frac{1}{3}$ is not a factor of the polynomial

5a. $f(x) = 3x^4 - 7x^3 - 20x^2 + 30x + 36$ Back to Problem 5.

Factors of 36: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

Factors of 3: 1, 3

Using 1 as a denominator: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

Using 3 as a denominator: $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

Possible rational zeros (roots): $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm 1, \pm \frac{4}{3}, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9 \ \pm 12, \pm 18, \pm 36$

Trying 1:

Coeff of $3x^4 - 7x^3 - 20x^2 + 30x + 36$

3	– 7	– 20	30	36	1
	3	– 4	– 24	6	
3	– 4	– 24	6	42	

NOTE: $f(1) = 42 \neq 0 \Rightarrow x - 1$ is not a factor of the polynomial f and 1 is not a zero (root) of f.

Trying -1:

$$\begin{array}{rrrrr|l} \multicolumn{5}{c}{\overbrace{\hspace{6cm}}^{Coeff\ of\ 3x^4 - 7x^3 - 20x^2 + 30x + 36}} & \\ 3 & -7 & -20 & 30 & 36 & \underline{-1} \\ & -3 & 10 & 10 & -40 & \\ \hline 3 & -10 & -10 & 40 & -4 & \end{array}$$

NOTE: $f(-1) = -4 \neq 0 \Rightarrow x + 1$ is not a factor of f and -1 is not a zero (root) of f.

Trying 2:

$$\begin{array}{rrrrr|l} \multicolumn{5}{c}{\overbrace{\hspace{6cm}}^{Coeff\ of\ 3x^4 - 7x^3 - 20x^2 + 30x + 36}} & \\ 3 & -7 & -20 & 30 & 36 & \underline{2} \\ & 6 & -2 & -44 & -28 & \\ \hline 3 & -1 & -22 & -14 & 8 & \end{array}$$

NOTE: $f(2) = 8 \neq 0 \Rightarrow x - 2$ is not a factor of the polynomial f and 2 is not a zero (root) of f.

Trying -2:

$$\begin{array}{rrrrr|l} \multicolumn{5}{c}{\overbrace{\hspace{6cm}}^{Coeff\ of\ 3x^4 - 7x^3 - 20x^2 + 30x + 36}} & \\ 3 & -7 & -20 & 30 & 36 & \underline{-2} \\ & -6 & 26 & -12 & -36 & \\ \hline 3 & -13 & 6 & 18 & 0 & \end{array}$$

NOTE: $f(-2) = 0 \Rightarrow x + 2$ is a factor of the polynomial f and -2 is a zero (root) of f.

NOTE: The third row in the synthetic division gives us the coefficients of the other factor starting with x^3. Thus, the other factor is $3x^3 - 13x^2 + 6x + 18$.

Thus, we have that $f(x) = 3x^4 - 7x^3 - 20x^2 + 30x + 36 =$

$(x + 2)(3x^3 - 13x^2 + 6x + 18)$.

Note that the remaining zeros of the polynomial f must also be zeros (roots) of the quotient polynomial $q(x) = 3x^3 - 13x^2 + 6x + 18$. We will use this polynomial to find the remaining zeros (roots) of the polynomial *f*, including another zero (root) of -2.

Trying -2 again:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{\hspace{6cm}}^{\textit{Coeff of } 3x^3 - 13x^2 + 6x + 18}} & \\ 3 & -13 & 6 & 18 & \underline{-2} \\ & -6 & 38 & -88 & \\ \hline 3 & -19 & 44 & -70 & \end{array}$$

NOTE: $q(-2) = -70 \neq 0 \Rightarrow x + 2$ is not a factor of the quotient polynomial *q*. Thus, the factor $x + 2$ only occurs once in the factorization of the polynomial *f* and -2 is a zero (root) of multiplicity one for the polynomial *f*.

NOTE: By the Bound Theorem, -2 is a lower bound for the negative zeros (roots) of the quotient polynomial $q(x) = 3x^3 - 13x^2 + 6x + 18$ since we alternate from **positive** 3 to **negative** 19 to **positive** 44 to **negative** 70 in the third row of the synthetic division. Thus, the remaining negative numbers in our list of possible rational zeros (roots) will not be zeros (roots) of the polynomial *q* nor *f*.

Trying 3:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{\hspace{6cm}}^{\textit{Coeff of } 3x^3 - 13x^2 + 6x + 18}} & \\ 3 & -13 & 6 & 18 & \underline{3} \\ & 9 & -12 & -18 & \\ \hline 3 & -4 & -6 & 0 & \end{array}$$

NOTE: $q(3) = 0 \Rightarrow x - 3$ is a factor of the quotient polynomial q and the polynomial *f*. Thus, 3 is a zero (root) of *q* and *f*.

Thus, $q(x) = 3x^3 - 13x^2 + 6x + 18 = (x - 3)(3x^2 - 4x - 6)$

Thus, $f(x) = 3x^4 - 7x^3 - 20x^2 + 30x + 36 =$
$(x + 2)(3x^3 - 13x^2 + 6x + 18) = (x + 2)(x - 3)(3x^2 - 4x - 6)$

Now, we can try to find a factorization for the quadratic expression $3x^2 - 4x - 6$. However, it does not factor.

Thus, we have that $3x^4 - 7x^3 - 20x^2 + 30x + 36 =$
$(x + 2)(x - 3)(3x^2 - 4x - 6)$.

Thus, $3x^4 - 7x^3 - 20x^2 + 30x + 36 = 0 \Rightarrow$

$(x + 2)(x - 3)(3x^2 - 4x - 6) = 0 \Rightarrow x = -2, \; x = 3,$

$3x^2 - 4x - 6 = 0$

We will need to use the Quadratic Formula to solve $3x^2 - 4x - 6 = 0$.

Thus, $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{16 - 4(3)(-6)}}{6} =$

$\dfrac{4 \pm \sqrt{4(4 + 18)}}{6} = \dfrac{4 \pm 2\sqrt{22}}{6} = \dfrac{2 \pm \sqrt{22}}{3}$

Answer: Zeros (Roots): $-2, \; 3, \; \dfrac{2 \pm \sqrt{22}}{3}$

Factorization: $(x + 2)(x - 3)(3x^2 - 4x - 6)$

5b. $g(x) = 2x^4 - 8x^3 + 15x^2 - 28x + 28$ Back to Problem 5.

Factors of 28: $\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

Factors of 2: 1, 2

Using 1 as a denominator: $\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

Using 2 as a denominator: $\pm \frac{1}{2}, \pm \frac{7}{2}$

Possible rational zeros (roots): $\pm \frac{1}{2}, \pm 1, \pm 2, \pm \frac{7}{2}, \pm 4, \pm 7, \pm 14, \pm 28$

Trying 1:

$$\begin{array}{rrrrr|l} \multicolumn{5}{c}{\overbrace{\hspace{6cm}}^{Coeff\ of\ 2x^4 - 8x^3 + 15x^2 - 28x + 28}} & \\ 2 & -8 & 15 & -28 & 28 & 1 \\ & 2 & -6 & 9 & -19 & \\ \hline 2 & -6 & 9 & -19 & 9 & \end{array}$$

NOTE: $g(1) = 9 \neq 0 \Rightarrow x - 1$ is not a factor of the polynomial g and 1 is not a zero (root) of g.

Trying -1:

$$\begin{array}{rrrrr|l} \multicolumn{5}{c}{\overbrace{\hspace{6cm}}^{Coeff\ of\ 2x^4 - 8x^3 + 15x^2 - 28x + 28}} & \\ 2 & -8 & 15 & -28 & 28 & -1 \\ & -2 & 10 & -25 & 53 & \\ \hline 2 & -10 & 25 & -53 & 81 & \end{array}$$

NOTE: $g(-1) = 81 \neq 0 \Rightarrow x + 1$ is not a factor of the polynomial g and -1 is not a zero (root) of g.

NOTE: By the Bound Theorem, -1 is a lower bound for the negative zeros (roots) of the polynomial g since we alternate from **positive** 2 to **negative** 10 to **positive** 25 to **negative** 53 to **positive** 81 in the third row of the synthetic division. Thus, the remaining negative numbers in our list of possible rational zeros (roots) will not be zeros (roots) of the polynomial *g*.

Trying 2:

$$\begin{array}{rrrrr|l} \multicolumn{5}{c}{\overbrace{\hspace{6cm}}^{Coeff\ of\ 2x^4 - 8x^3 + 15x^2 - 28x + 28}} & \\ 2 & -8 & 15 & -28 & 28 & \underline{2} \\ & 4 & -8 & 14 & -28 & \\ \hline 2 & -4 & 7 & -14 & 0 & \end{array}$$

NOTE: $g(2) = 0 \Rightarrow x - 2$ is a factor of the polynomial g and 2 is a zero (root) of g.

The third row in the synthetic division gives us the coefficients of the other factor starting with x^3. Thus, the other factor is $2x^3 - 4x^2 + 7x - 14$.

Thus, we have that $g(x) = 2x^4 - 8x^3 + 15x^2 - 28x + 28 =$ $(x - 2)(2x^3 - 4x^2 + 7x - 14)$.

Note that the remaining zeros of the polynomial g must also be zeros (roots) of the quotient polynomial $q(x) = 2x^3 - 4x^2 + 7x - 14$. We will use this polynomial to find the remaining zeros (roots) of the polynomial *g*, including another zero (root) of 2.

Trying 2:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{\hspace{4.5cm}}^{Coeff\ of\ 2x^3 - 4x^2 + 7x - 14}} & \\ 2 & -4 & 7 & -14 & \underline{2} \\ & 4 & 0 & 14 & \\ \hline 2 & 0 & 7 & 0 & \end{array}$$

NOTE: $q(2) = 0 \Rightarrow x - 2$ is a factor of the quotient polynomial q and is a second factor of the polynomial *g*. Thus, the factor $x - 2$ occurs twice in the factorization of the polynomial *g* and 2 is a zero (root) of multiplicity two for the polynomial *g*.

Thus, $q(x) = 2x^3 - 4x^2 + 7x - 14 = (x - 2)(2x^2 + 7)$

Thus, $g(x) = 2x^4 - 8x^3 + 15x^2 - 28x + 28 =$
$(x - 2)(2x^3 - 4x^2 + 7x - 14) = (x - 2)(x - 2)(2x^2 + 7) =$
$(x - 2)^2(2x^2 + 7)$.

The quadratic expression $2x^2 + 7$ does not factor.

Thus, we have that $2x^4 - 8x^3 + 15x^2 - 28x + 28 = (x - 2)^2(2x^2 + 7)$.

Thus, $2x^4 - 8x^3 + 15x^2 - 28x + 28 = 0 \Rightarrow$

$(x - 2)^2(2x^2 + 7) = 0 \Rightarrow (x - 2)^2 = 0, \; 2x^2 + 7 = 0$.

$(x - 2)^2 = 0 \Rightarrow x - 2 = 0 \Rightarrow x = 2$

$2x^2 + 7 = 0 \Rightarrow 2x^2 = -7 \Rightarrow x^2 = -\frac{7}{2} \Rightarrow x = \pm i\sqrt{\frac{7}{2}} = \pm \frac{\sqrt{14}}{2} i$

Answer: Zeros (Roots): 2, $\pm \frac{\sqrt{14}}{2} i$

Back to Problem 5.

Factorization: $(x - 2)^2(2x^2 + 7)$