In-Class Problems 11 for Wednesday, February 28

These problems are from Pre-Class Problems 11.

You can go to the solution for each problem by clicking on the problem number.

Find the zeros (roots) and their multiplicities. Discuss the implication of the multiplicity on the graph of the polynomial. Determine the sign of the infinity that the polynomial values approaches as x approaches positive infinity and negative infinity. Use this information to determine the number of relative (local) extremum points (turning points) that the graph of the polynomial has. Sketch a graph of the polynomial.

1. $f(x) = x^3 - 8x^2 + 16x$

2. $g(x) = 2x(6 - x)(3x - 5)^2$

SOLUTIONS:

1. $f(x) = x^3 - 8x^2 + 16x$ Back to Problem 1.

Zeros (Roots) of f: $f(x) = 0 \Rightarrow x^3 - 8x^2 + 16x = 0 \Rightarrow$

$x(x^2 - 8x + 16) = 0 \Rightarrow x(x - 4)^2 = 0 \Rightarrow x = 0, \; x = 4$

NOTE: Since the factor x produces the zero (root) of 0, its multiplicity is one. Since the factor $(x - 4)^2$ produces the zero (root) of 4, its multiplicity is two.

Zero (Root)	**Multiplicity**	**Implication for the Graph of the Polynomial**
0	1	Crosses the x-axis at $(0, 0)$
4	2	Touches the x-axis at $(4, 0)$

For infinitely large values of x, $f(x) \approx x^3$

As $x \to \infty$, $x^3 \to \infty$. Thus, $f(x) \approx x^3 \to \infty$.

As $x \to -\infty$, $x^3 \to -\infty$. Thus, $f(x) \approx x^3 \to -\infty$.

The polynomial is neither even nor odd: $f(x) = x^3 - 8x^2 + 16x \Rightarrow$

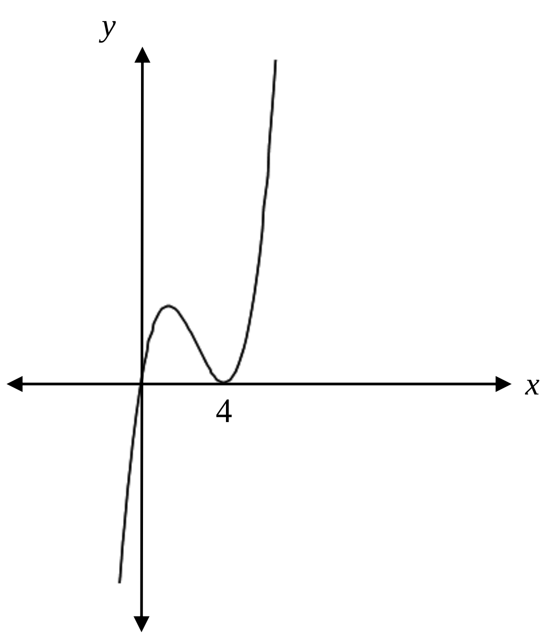
$f(-x) = (-x)^3 - 8(-x)^2 + 16(-x) = -x^3 - 8x^2 - 16x$

Thus, $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

Thus, the graph of the polynomial is not symmetric with respect to the y-axis nor the origin.

Back to Problem 1.

Here is the information about the zeros (roots) of the polynomial and the graph of the polynomial as $x \to \infty$ and as $x \to -\infty$.



Back to Problem 1.

NOTE: Since the degree of the polynomial is 3, then the graph of the polynomial can have at most 2 local extremum points (turning points).

NOTE: Since the graph touches at the point $(4, 0)$, then there is a local extremum point (turning point) at this point. Since the graph of the continuous polynomial must cross the *x*-axis at the origin, then there is another local extremum point (turning point) whose *x*-coordinate is between 0 and 4.

Thus, the graph of the polynomial has two local extremum points (turning points).

2. $g(x) = 2x(6 - x)(3x - 5)^2$ Back to Problem 2.

Zeros (Roots) of g: $g(x) = 0 \Rightarrow 2x(6 - x)(3x - 5)^2 = 0 \Rightarrow$

$x = 0, \; x = 6, \; x = \dfrac{5}{3}$

NOTE: Since the factor *x* produces the zero (root) of 0, its multiplicity is one. Since the factor $6 - x$ produces the zero (root) of 6, its multiplicity is one. Since the factor $(3x - 5)^2$ produces the zero (root) of $\frac{5}{3}$, its multiplicity is two.

Zero (Root)	**Multiplicity**	**Implication for the Graph of the Polynomial**
0	1	Crosses the *x*-axis at $(0, 0)$
$\frac{5}{3}$	2	Touches the *x*-axis at $\left(\frac{5}{3}, 0\right)$
6	1	Crosses the *x*-axis at $(6, 0)$

For infinitely large values of *x*, $g(x) \approx 2x(-x)(3x)^2 = -18x^4$

As $x \to \infty$, $x^4 \to \infty$. Thus, since $-18 < 0$, then $g(x) \approx -18x^4 \to -\infty$.

As $x \to -\infty$, $x^4 \to \infty$. Thus, since $-18 < 0$, then $g(x) \approx -18x^4 \to -\infty$.

The polynomial is neither even nor odd:

$g(x) = 2x(6 - x)(3x - 5)^2 \Rightarrow$

$g(-x) = 2(-x)(6 + x)[3(-x) - 5]^2 =$

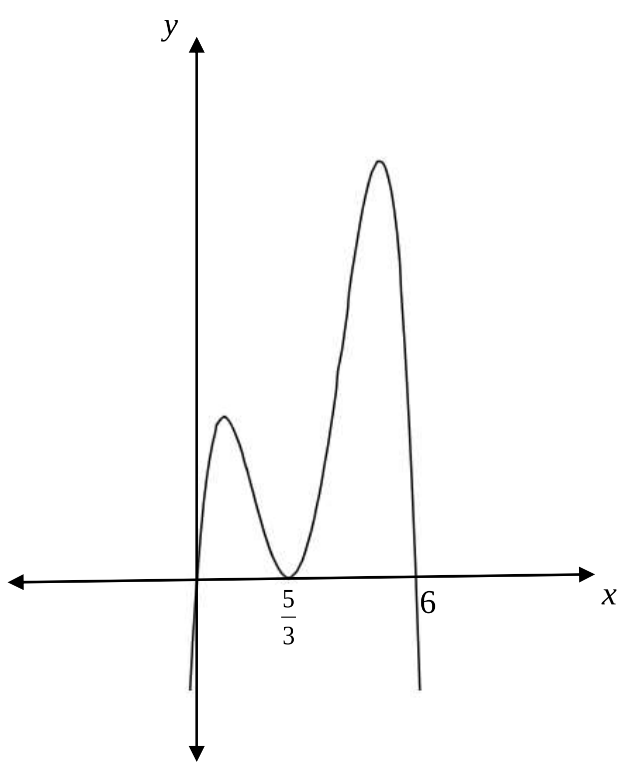
$-2x(6 + x)[-(3x + 5)]^2 = -2x(x + 6)(3x + 5)^2$

Thus, $g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$.

Thus, the graph of the polynomial is not symmetric with respect to the *y*-axis nor the origin.

Back to Problem 2.

Here is the information about the zeros (roots) of the polynomial and the graph of the polynomial as $x \to \infty$ and as $x \to -\infty$.



Back to Problem 2.

NOTE: Since the degree of the polynomial is 4, then the graph of the polynomial can have at most 3 local extremum points (turning points).

NOTE: The graph of the continuous polynomial must cross the *x*-axis at the origin. Then the graph must touch the *x*-axis at the point $\left(\frac{5}{3}, 0\right)$. In order for this to happen, there must be a local extremum point (turning point) whose *x*-coordinate is between 0 and $\frac{5}{3}$. Since the graph touches at the point $\left(\frac{5}{3}, 0\right)$, then there is a local extremum point (turning point) at this point. Then the graph of the polynomial must cross the *x*-axis at the point $(6, 0)$. In order for this to happen, there must be a local extremum point (turning point) whose *x*-coordinate is between $\frac{5}{3}$ and 6.

Thus, the graph of the polynomial has three local extremum points (turning points).

Back to Problem 2.