Solutions for In-Class Problems 10 for Monday, February 26

These problems are from Pre-Class Problems 10.

You can go to the solution for each problem by clicking on the problem letter.

1. If $f(x) = \sqrt{2x + 13}$ and $g(x) = 3x^2 - 5x - 27$, then find

   a. $(f \circ g)(-2)$ b. $(g \circ f)(-2)$

2. If $f(x) = 9 - x^2$ and $g(x) = 4x - 7$, then find

   a. $(f \circ g)(x)$ b. $(g \circ f)(x)$

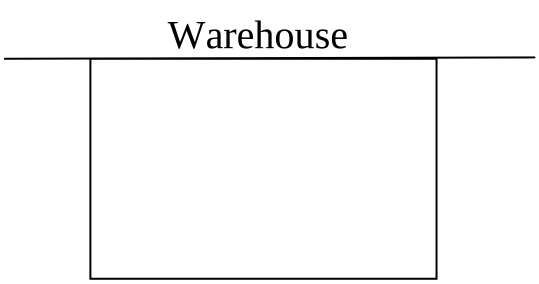
3. Given the function h, find functions f and g such that $h = f \circ g$.

   a. $h(x) = \dfrac{8}{3x + 16}$ b. $h(x) = \sqrt[5]{\dfrac{5x + 3}{x + 2}}$

4. Write the following quadratic functions in vertex form by completing the square. Then a. identify the vertex, b. determine whether the graph of the parabola opens upward or downward, c. determine the x-intercept(s), d. determine the y-intercept, e. sketch the graph of the function, f. determine the axis of symmetry, g. determine the maximum or minimum value of the function, h. determine the range of the function.

   a. $f(x) = -x^2 + 8x - 14$ b. $g(x) = 3x^2 + 2x - 8$

5. A company wants to use fencing to enclose a rectangular region next to a warehouse. If they have 400 yards of fencing and they do not fence in the side next to the warehouse, what is the largest area that they can enclose?



SOLUTIONS:

1. If $f(x) = \sqrt{2x + 13}$ and $g(x) = 3x^2 - 5x - 27$, then find

$f(-2) = \sqrt{9} = 3$

$g(-2) = 12 + 10 - 27 = -5$

1a. $(f \circ g)(-2) = f(g(-2)) = f(-5) = \sqrt{3}$

Answer: $\sqrt{3}$ Back to Problem 1.

1b. $(g \circ f)(-2) = g(f(-2)) = g(3) = 27 - 15 - 27 = -15$

Answer: -15 Back to Problem 1.

2. $f(x) = 9 - x^2$ and $g(x) = 4x - 7$

2a. $(f \circ g)(x) = f(g(x)) = f(4x - 7) = 9 - (4x - 7)^2 =$

$9 - (16x^2 - 56x + 49) = 9 - 16x^2 + 56x - 49 =$

$-16x^2 + 56x - 40$

Answer: $-16x^2 + 56x - 40$ Back to Problem 2.

2b. $(g \circ f)(x) = g(f(x)) = g(9 - x^2) = 4(9 - x^2) - 7 =$

$36 - 4x^2 - 7 = 29 - 4x^2$

Answer: $29 - 4x^2$ Back to Problem 2.

3a. $h(x) = \dfrac{8}{3x + 16}$

Let $f(x) = \dfrac{8}{x}$ and let $g(x) = 3x + 16$. Then $h(x) = (f \circ g)(x) =$

$f(g(x)) = f(3x + 16) = \dfrac{8}{3x + 16}$.

Answer: $f(x) = \dfrac{8}{x}$, $g(x) = 3x + 16$ Back to Problem 3.

3b. $h(x) = \sqrt[5]{\dfrac{5x + 3}{x + 2}}$

Let $f(x) = \sqrt[5]{x}$ and let $g(x) = \dfrac{5x + 3}{x + 2}$. Then $h(x) = (f \circ g)(x) =$

$f(g(x)) = f\left(\dfrac{5x + 3}{x + 2}\right) = \sqrt[5]{\dfrac{5x + 3}{x + 2}}$.

Answer: $f(x) = \sqrt[5]{x}$, $g(x) = \dfrac{5x + 3}{x + 2}$ Back to Problem 3.

4a. $f(x) = -x^2 + 8x - 14$ Back to Problem 4.

$y = -x^2 + 8x - 14$

$y + 14 \underline{- 16} = -(x^2 - 8x + \underline{16}) \Rightarrow y - 2 = -(x - 4)^2$

a. **Vertex:** $(4, 2)$

b. $ax^2 = -x^2 \Rightarrow a = -1 \Rightarrow$ **parabola opens downward**

c. *x*-intercept(s): Set $y = 0 \Rightarrow -2 = -(x - 4)^2 \Rightarrow (x - 4)^2 = 2$

$\Rightarrow x - 4 = \pm\sqrt{2} \Rightarrow x = 4 \pm \sqrt{2}$

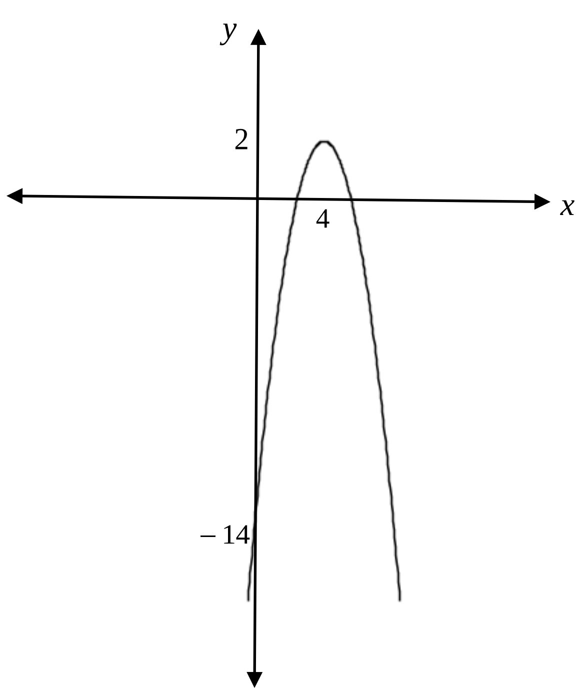
Answer: $(4 \pm \sqrt{2}, 0)$

d. *y*-intercept: Set $x = 0 \Rightarrow y = -14$

Answer: $(0, -14)$

Back to Problem 4.

e.



f. axis of symmetry: Since the vertex of the parabola is the point $(4,\ 2)$, then the axis of symmetry is the vertical line $x = 4$.

Answer: $x = 4$

g. Since the parabola opens downward and the vertex of the parabola is the point $(4,\ 2)$, then the function only has a maximum value of 2.

Answer: 2 Back to Problem 4.

h. Range: $(-\infty,\ 2]$

4b. $g(x) = 3x^2 + 2x - 8$

$y = 3x^2 + 2x - 8$

$$y + 8 + \underline{\frac{1}{3}} = 3\left(x^2 + \frac{2}{3}x + \underline{\frac{1}{9}}\right) \Rightarrow y + \frac{25}{3} = 3\left(x + \frac{1}{3}\right)^2$$

a. **Vertex:** $\left(-\frac{1}{3},\ -\frac{25}{3}\right)$ Back to Problem 4.

b. $ax^2 = 3x^2 \Rightarrow a = 3 \Rightarrow$ **parabola opens upward**

c. x-intercept(s): Set $y = 0 \Rightarrow \dfrac{25}{3} = 3\left(x + \dfrac{1}{3}\right)^2 \Rightarrow$

$$\frac{25}{9} = \left(x + \frac{1}{3}\right)^2 \Rightarrow x + \frac{1}{3} = \pm\frac{5}{3} \Rightarrow x = -\frac{1}{3} \pm \frac{5}{3} = \frac{-1 \pm 5}{3}$$

$$x = -2, \ \frac{4}{3}$$

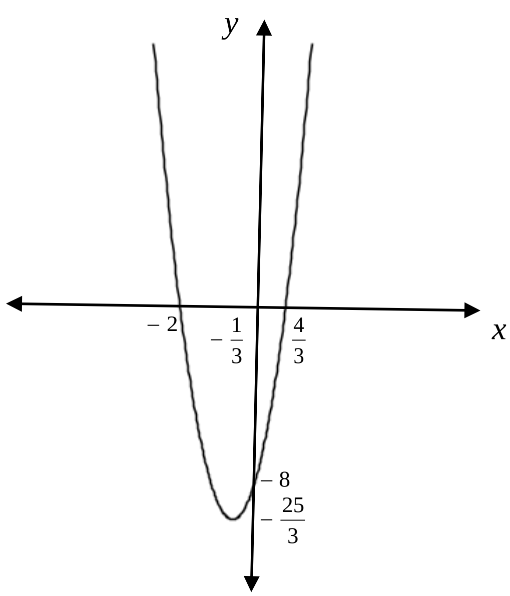
Answer: $(-2,\ 0)$, $\left(\dfrac{4}{3},\ 0\right)$

Back to Problem 4.

d. y-intercept: Set $x = 0 \Rightarrow y = -8$

Answer: $(0,\ -8)$

e.



f. axis of symmetry: Since the vertex of the parabola is the point $\left(-\dfrac{1}{3},\ -\dfrac{25}{3}\right)$, then the axis of symmetry is the vertical line $x = -\dfrac{1}{3}$.

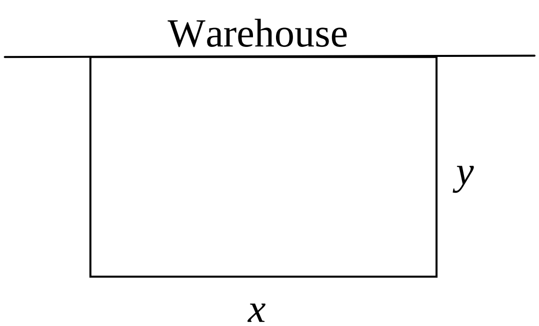
Answer: $x = -\frac{1}{3}$ Back to Problem 4.

g. Since the parabola opens upward and the vertex of the parabola is the point $\left(-\frac{1}{3}, -\frac{25}{3}\right)$, then the function only has a minimum value of $-\frac{25}{3}$.

Answer: $-\frac{25}{3}$

h. Range: $\left[-\frac{25}{3}, \infty\right)$ Back to Problem 4.

5. A company wants to use fencing to enclose a rectangular region next to a warehouse. If they have 400 yards of fencing and they do not fence in the side next to the warehouse, what is the largest area that they can enclose?



$A = xy$

NOTE: The amount of fencing that would be used to make the enclosure according to the diagram is $x + 2y$.

Since the company has 400 yards of fencing, then $x + 2y = 400$. Solving for *x*, we have that $x = 400 - 2y$.

Thus, $A = xy$ and $x = 400 - 2y \Rightarrow A = (400 - 2y)y$

$A = (400 - 2y)y = 400y - 2y^2 = -2y^2 + 400y$

Completing the square on $A = -2y^2 + 400y$:

$$A \underline{- 20000} = -2(y^2 - 200y + \underline{10000}) \Rightarrow A - 20000 = -2(y - 100)^2$$

$\downarrow$ *Half*

100

$\downarrow$ *Square*

10000

NOTE: $-2 \cdot 10000 = -20000$. Thus, we subtracted 20000 from the right side of the first equation above. So, to keep the equation equivalent, we subtract 20000 from the left side of this equation.

The vertex of this parabola is of the form (y, A) and the vertex is $(100, 20000)$. Since $ay^2 = -2y^2 \Rightarrow a = -2 < 0$, then the parabola opens downward. Thus, A has a maximum of 20,000 which occurs when $y = 100$. That is, the area of the rectangular enclosure has a maximum area of 20,000 square yards occurring when $y = 100$ yards.

Answer: 20,000 square yards Back to Problem 5.

NOTE: Since $x = 400 - 2y$ and $y = 100$, then $x = 400 - 200 = 200$. Thus, the dimensions of the rectangular enclosure are 200 yards by 100 yards.