1 PART I: Physical Applications

1. *A thin rod from $x = 0$ to $x = L$ has density $\rho(x) = x$.*

a) What is the mass of the rod?

b) If the rod has mass 40, what is L?

2. *A thin rod from $x = 0$ to $x = 10$ has density*

$$\rho(x) = \begin{cases} 4, 0 \leq x \leq a \\ 4x^3, a \leq x \leq 10 \end{cases}$$

a) Find the mass of the rod in terms of a.

b) What value of a ensures that the mass of the portion of the rod left of $x = a$ has the same mass as the portion of the rod right of a

3. *A spring with spring constant k requires $80N$ to stretch $2m$.*

a) How much work is required to stretch the spring?

b) How much work is required to stretch it an additional $2m$?

c) Suppose the spring (which is now 4 meters from equilibrium position) is stretched an additional a meters. What should a be so that the total work stretching the spring to $4 + a$ m is $500J$?

4. *A cylindrical tank has base radius $5m$ and height 20 m. The tank is filled up to a hight b with liquid X ($\rho = 5000kg/m^3$)and then filled the rest of the way with liquid Y ($\rho = 2000kg/m^3$). Assuming that the liquids do not mix, find a value for b so the work required to pump liquid X from the tank is equal to the amount of work required to pump liquid Y from the tank.*

2 PART II: u-substitutions

1. $\int 3x^2 \sin(4x^3)dx$
2. $\int_0^1 x^2 e^{4x^3+1}dx$

3. $\int \frac{4x+6}{3x^2+9x}dx$
4. $\int 5x^8(14x^9-1)^6dx$
5. $\int_{\pi/4}^{\pi/2} \frac{\cos(\sqrt{x})}{2\sqrt{x}}dx$
6. $\int_0^2 \frac{1}{2x+4}dx$
7. $\int_{\sqrt{\pi/3}}^{\sqrt{\pi/6}} 2x\sec(4x^2)\tan(4x^2)dx$

3 PART II: Various Integrals

1. $\int x(x^3+1)^2dx$
2. $\int x^2(x^3+1)^5dx$
3. $\int \frac{5x-5x^2}{3x^3}dx$
4. $\int \frac{1}{x\ln x}dx$
5. $\int \frac{2x^3-4x^2+8x-1}{2x+1}dx$
6. $\int \frac{3x+4\sqrt{x}}{9x^2+16\sqrt{x^3}}dx$
7. $\int 14e^x\sec^2(e^x)dx$

4 PART III: Integration by parts

1. $\int 4x\cos 5xdx$
2. $\int x\ln xdx$
3. $\int_0^{\pi} e^{2x}\cos 4xdx$

4. $\int \arcsin x dx$
5. $\int (\ln x)^2 dx$
6. $\int x^2 \sin 4x dx$
7. $\int_0^{\pi} \sin 2x \cos 3x dx$