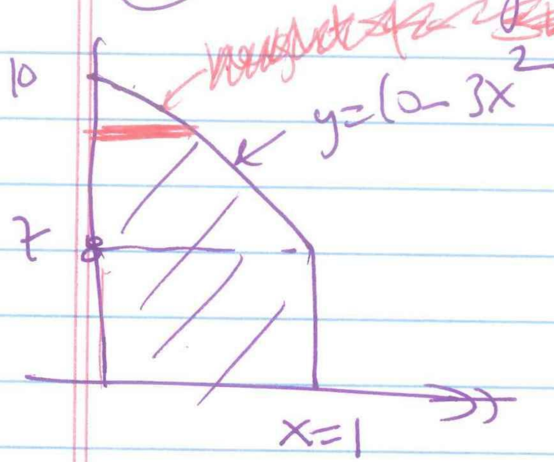
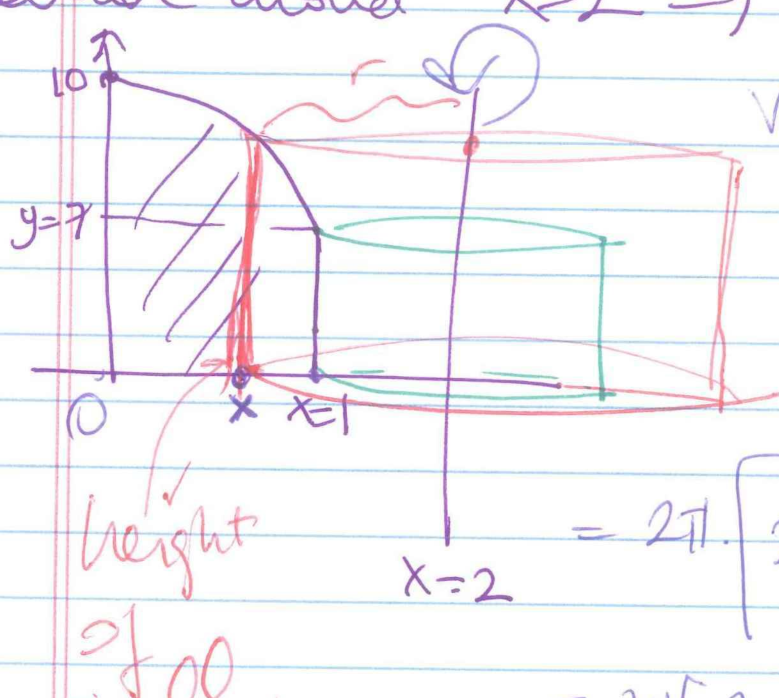
WS III — Partial Slns:

① with cylindrical shells:



①. Revolve around x=2 ⟹



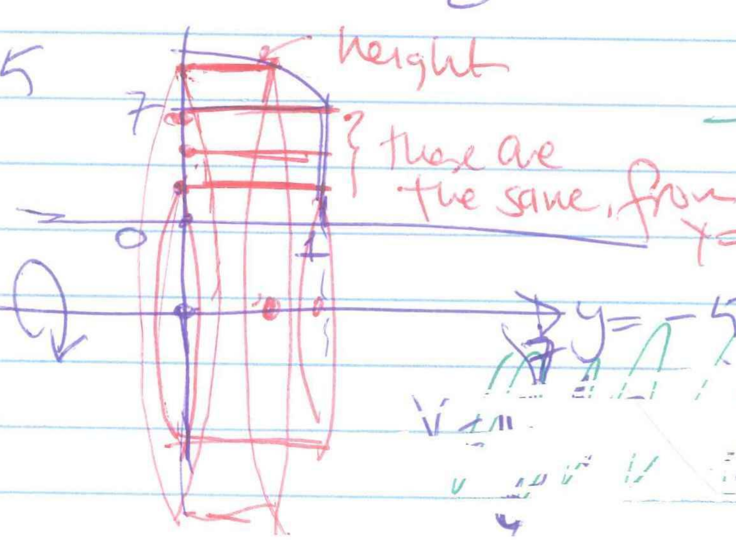
$$V=2\pi\int_0^1 \overbrace{(2-x)}^{r}\,\overbrace{(10-3x^2)}^{h}\,dx$$

$$=2\pi\int_0^1 (20-6x^2-10x+3x^3)\,dx$$

$$=2\pi\left[20x-3x^3-5x^2+\frac{3x^4}{4}\right]_0^1$$

$$=2\pi\left[20-3-5+\frac{3}{4}\right]=\frac{81}{2}\pi.$$

② Around y = −5



→ in the back.

$$V = 2\pi \Big[\underbrace{\int_0^7 (5+y)\cdot 1\, dy}_{①} + \underbrace{\int_7^{10} (5+y)\cdot \sqrt{\frac{10-y}{3}}\, dy}_{②} \Big]$$

② Let $10-y=u \Rightarrow -dy = du$. $\left(u = 10-y \Rightarrow y = 10-u \right)$

Thus, Also switch this

$$② = -\frac{1}{\sqrt{3}} \int_3^0 \underbrace{(5+10-u)}_{(15-u)} \cdot \sqrt{u}\, du \qquad \begin{aligned} y=7 &\Rightarrow u=3 \\ y=10 &\Rightarrow u=0 \end{aligned}$$

$$= -\frac{1}{\sqrt{3}} \left(-\int_0^3 (15-u)\cdot u^{1/2}\, du \right) = \frac{1}{\sqrt{3}} \int_0^3 \left(15u^{1/2} - u^{3/2}\right) du$$

$$= \frac{1}{\sqrt{3}} \left(15\frac{u^{3/2}}{3/2} - \frac{2}{5} u^{5/2} \right) \Big|_0^3$$

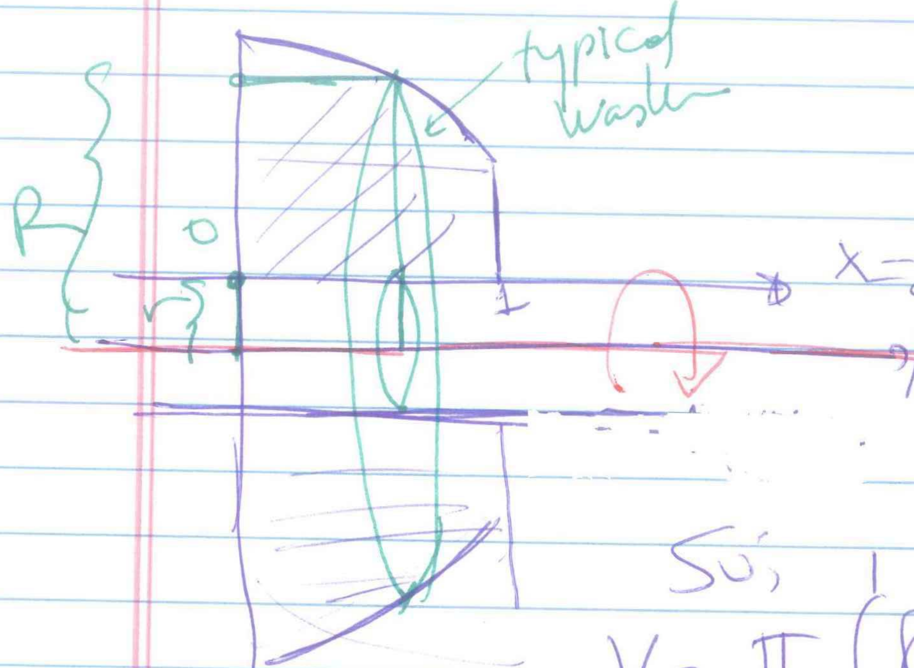
$$= \frac{1}{\sqrt{3}} \left[10\cdot(3)^{3/2} - \frac{2}{5}\cdot(3)^{5/2} \right] = \frac{1}{\sqrt{3}} \left[30\sqrt{3} - \frac{18\sqrt{3}}{5} \right]$$

$$= 30 - \frac{18}{5} = \frac{132}{5}.$$

$$① = \left(5y + \frac{y^2}{2}\right)\Big|_0^7 = 35 + \frac{49}{2} = \frac{119}{2}.$$

$\boxed{\text{Su}}$ $V = 2\pi \left[\frac{119}{2} + \frac{132}{5} \right] = \frac{859\pi}{5}$

It's easier with WASHER:



$\boxed{y=-5}$ (Axis of revolution)

$R = (5 + 10 - 3x^2)$

$r = 5.$

So,

$$V = \pi \int_0^1 \left((15 - 3x^2)^2 - 5^2 \right) dx$$

(3)

$y = 1 + 5x^{3/2}$ $x = 1$ to $x = 5$

$y' = \frac{15}{2} x^{1/2}$

$(y')^2 = \frac{225}{4} x.$

So

$L = \int_0^4 \sqrt{1 + \frac{225}{4} x}\, dx =$

$u = 1 + \frac{225}{4} x$

$du = \frac{225}{4} dx$

So $\int (\ldots)\, dx = \int u^{1/2} \frac{4}{225} du$

$= \frac{4}{225} \cdot \frac{2}{3} \cdot u^{3/2} + C$

$= \frac{8}{775} \left(1 + \frac{225}{4} x\right) \Big|_0^4$

$= \frac{8}{775} \left[(1 + 225)^{3/2} - (1)^{3/2} \right]$

$= \frac{8}{775} \left[226^{3/2} - 1 \right]$

(4) $x = \frac{1}{6}\left[e^{3y} + e^{-3y}\right] = \frac{1}{6}e^{3y} + \frac{1}{6}e^{-3y}$

$x' = \frac{1}{2}e^{3y} - \frac{1}{2}e^{-3y}$

$(x')^2 = \frac{1}{4}e^{6y} - \frac{1}{2}\underbrace{(e^{3y} \cdot e^{-3y})}_{1} + \frac{1}{4}e^{-6y}$

$\Rightarrow \quad L = \int_0^1 \sqrt{1+(x')^2}\,dx = \int_0^1 \sqrt{1 + \frac{1}{4}e^{6y} - \frac{1}{2} + \frac{1}{4}e^{-6y}}\,dy$

$= \int_0^1 \sqrt{\frac{1}{4}e^{6y} + \frac{1}{2} + \frac{1}{4}e^{-6y}}\,dy = \int_0^1 \sqrt{\left(\frac{1}{2}e^{3y} + \frac{1}{2}e^{-3y}\right)^2}\,dy$

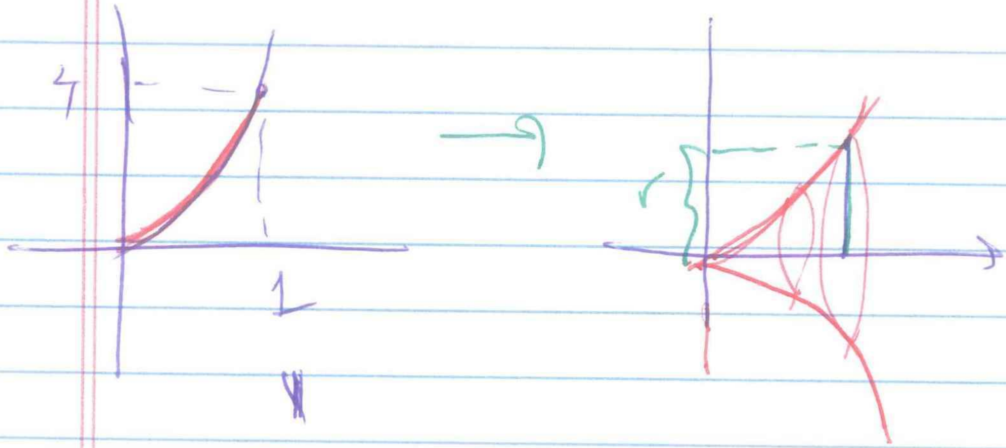
$= \int_0^1 \left(\frac{1}{2}e^{3y} + \frac{1}{2}e^{-3y}\right)dy$

$= \left[\frac{1}{6}e^{3y} - \frac{1}{6}e^{-3y}\right]_0^1 = \frac{1}{6}\left[(e^{3} - e^{-3}) - (e^{0} - e^{0})\right]$

$= \frac{1}{6}(e^{3} - e^{-3})$

(5) Find the area of the surface generated by revolving given curve about given axis.

(a) $y = 4x^3$, $x = 0$ to $x = 1$, about x



$$SA = \int_0^1 \sqrt{1 + f'(x)^2} \cdot (2\pi r)\, dx \qquad f(x) = 4x^3, \quad f'(x) = 12x^2$$

$$= 2\pi \int_0^1 \overset{r}{4x^3} \sqrt{1 + (144)x^4}\, dx \qquad 1 + 144x^4 = u, \quad (144)\cdot 4x^3\, dx = du$$

$$= \frac{2\pi}{144} \int \sqrt{u}\, du = \frac{\pi}{72} \frac{u^{3/2}}{3/2} = \frac{\pi}{108} u^{3/2}$$

$$\Rightarrow SA = \frac{\pi}{108} \left(1 + 144x^4\right)^{3/2} \Big|_0^1 = \frac{\pi}{108}\left[145 - 1\right] = \frac{72}{54}\pi$$

(5) $x=\sqrt{2y+1}$ from $x=1$ to $x=3$ about y-axis

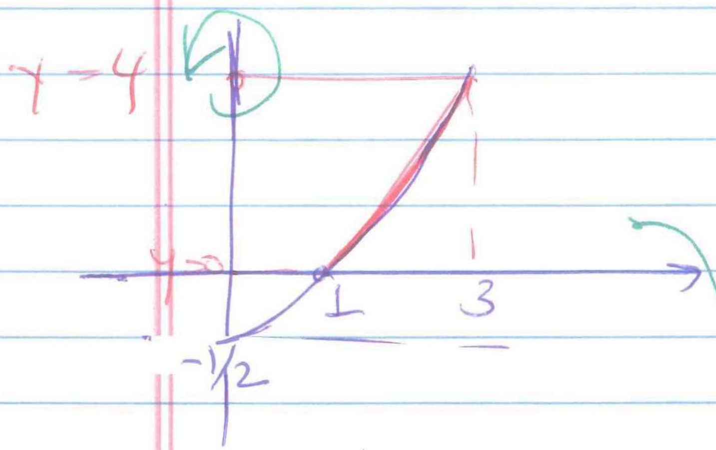
Function given as a function of y

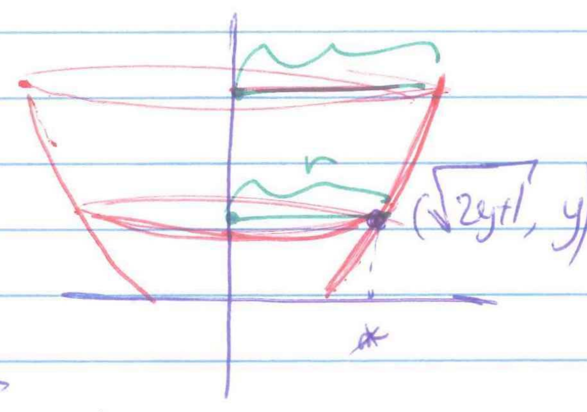
So, lets convert $x=1$ & $x=3$ to y

When $x=1$, $1=\sqrt{2y+1} \Rightarrow 2y=0 \Rightarrow \boxed{y=0}$

$x=3 \Rightarrow 3=\sqrt{2y+1} \Rightarrow 9=2y+1$

$\Rightarrow \boxed{y=4}$





$$SA = 2\pi \int_0^4 \sqrt{2y+1}\,\sqrt{1+\frac{1}{2y+1}}\,dy$$

$g(y)=\sqrt{2y+1}$

$g'(y) = \frac{\cancel{2}}{\cancel{2}\sqrt{2y+1}}$

$\Rightarrow (g'(y))^2 = \frac{1}{2y+1}$

$$= 2\pi \int_0^4 \cancel{\sqrt{2y+1}} \cdot \frac{\sqrt{2y+2}}{\cancel{\sqrt{2y+1}}}\,dy$$

$2y+2=u$

$2\,dy=du$

$y=0 \Rightarrow u=2$

$y=4 \Rightarrow u=10$

$$= \pi \int_2^{10} \sqrt{u}\,du = \pi \cdot \frac{2}{3}\left(u^{3/2}\right)\Big|_2^{10} = \frac{2\pi}{3}\left[10\sqrt{10}-2\sqrt{2}\right]$$