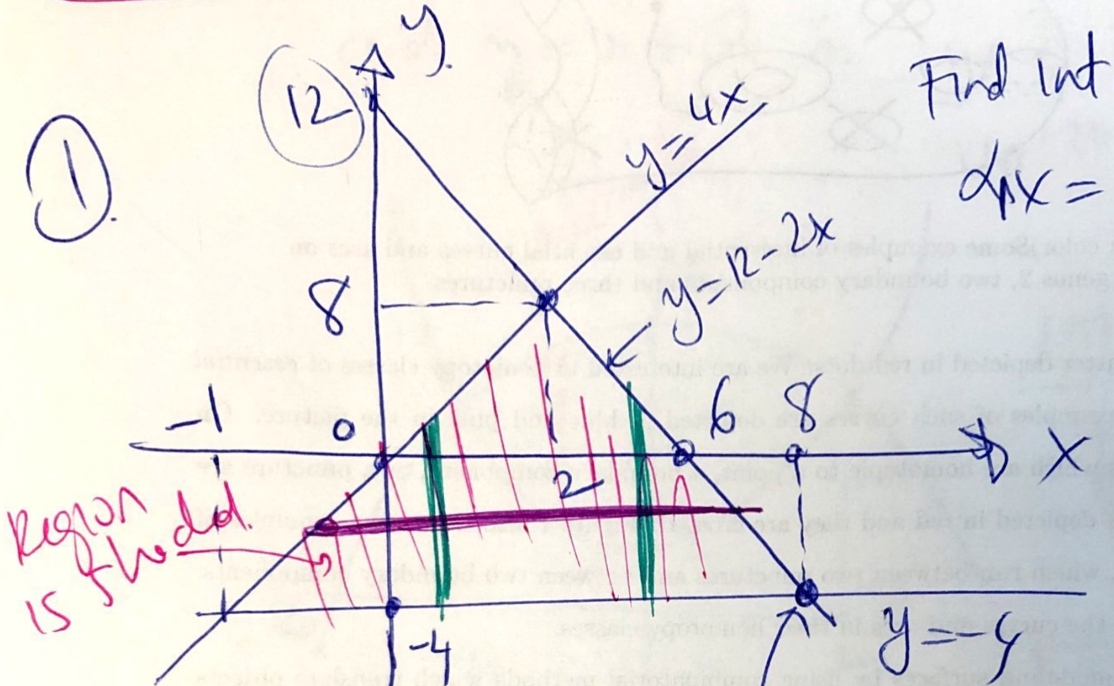
Partial Solutions to WS II

(1)



Find intersections

$4x = 12-2x \Rightarrow \boxed{x=2}$

$12-2x=-4 \Rightarrow 16=2x \Rightarrow x=8$

Draw a rectangle in the region along y axis.

(PURPLE LINE REPRESENTS IT)

$$\text{Area} = \int_{y=-4}^{8} \text{Height of the rectangle} \cdot dy = \int_{-4}^{8}\left[\left(6-\frac{y}{2}\right)-\frac{y}{4}\right]dy = 54$$

$y=12-2x \Rightarrow x=6-\frac{y}{2}$

If $y=4x$, $x=\frac{y}{4}$

If you try to do this along x axis

(GREEN LINES)

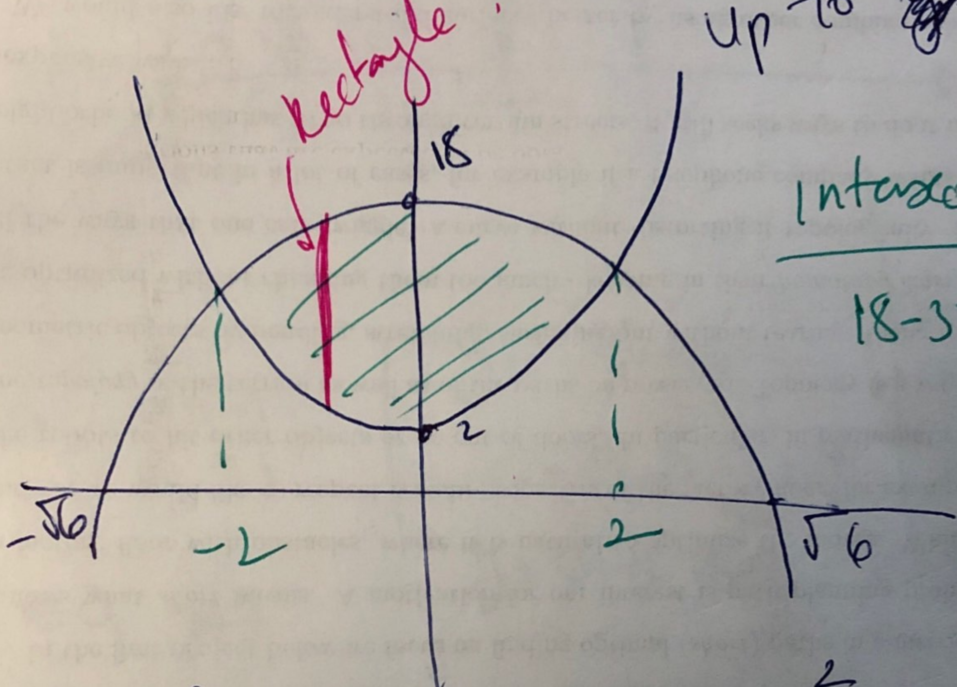
$$A=\int_{-1}^{2}\left[4x-(-4)\right]dx+\int_{2}^{8}(12-2x+4)\,dx$$

Scanned with CamScanner

(2) $y = 18 - 3x^2$ $\quad 18 - 3x^2 = 0 \Rightarrow 18 = 3x^2$

$\Rightarrow x = \pm\sqrt{6}$

and $y = x^2 + 2$ is $y = x^2$ shifted 2 units up to $\cancel{y} = 2$



Intersection pts

$18 - 3x^2 = x^2 + 2$

$\Downarrow$

$4x^2 = 16$

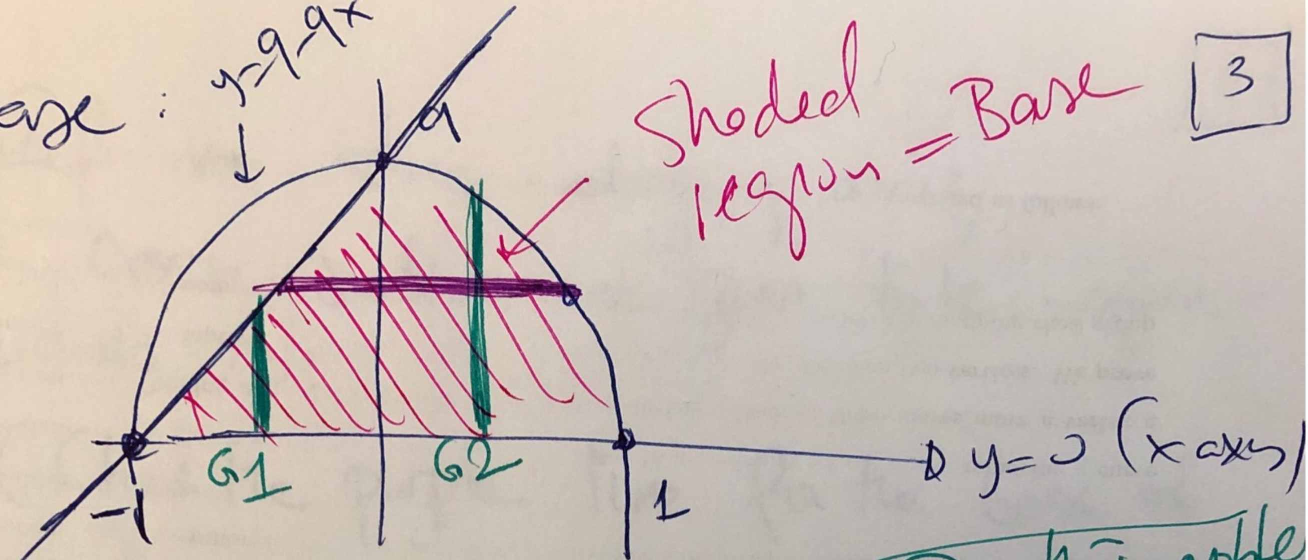
$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

$$\text{Area} = \int_{-2}^{2} \left[18 - 3x^2 - (x^2+2)\right] dx = 2\int_{0}^{2} (16 - 4x^2)\,dx$$

$$= 2\left(16x - \frac{4x^3}{3}\right)\Big|_0^2 = 2\left[32 - \frac{4}{3}\cdot 8\right] = \frac{128}{3}$$

4) Base : $y = 9 - 9x^2$



3

$y = 9 - 9x^2$

If $y = 0$, $x^2 = 1$
$x = \pm 1$

Also, If $x = 0$, $y = 9$

$y = 9 + 9x$
$x = 0 \Rightarrow y = 9$
$y = 0 \Rightarrow x = -1$

In this problem there is **no** revolution. Instead, you given cross-section

(c) If: Cross-Sections are squares perp. to x axis

Need to find dimensions of those squares!

So If base of square is changing along x axis: (green ~~line~~ line in the pic) There are 2 parts

$$V = \int_{-1}^{0} (\underbrace{9+9x-0}_{\text{length of Green 1}})^2\,dx + \int_{0}^{1} (\underbrace{9-9x^2-0}_{\text{length of green 2}})^2\,dx$$

(5) If we slice along y axis, the cross sections are given to be -again- squares, so:

(Check the purple line for the base of a square.)

$$V = \int_0^9 \Big[\underbrace{\sqrt{1-\tfrac{y}{9}} - \left(1-\tfrac{y}{9}\right)}_{\text{length of the purple line (base of square)}}\Big]^2 dy.$$

$y = 9 - 9x^2$

$\Downarrow$

$9x^2 = 9 - y$

$\Rightarrow x^2 = 1 - \frac{y}{9}$

$x = \pm\sqrt{1-\frac{y}{9}}$

Because we are working on RHS, we will take <u>positive</u> part

i.e. $x = \frac{\sqrt{9-y}}{3}$

<u>And</u>, $y = 9 - 9x \Rightarrow 9x = 9 - y$

$\Rightarrow x = 1 - \frac{y}{9}$

$\boxed{5}$

The integral:

$$V = \int_0^9 \left[\sqrt{1-\frac{y}{9}} - \left(1-\frac{y}{9}\right)\right]^2 dy$$ (Do the square!)

$$= \int_0^9 \left[\left(1-\frac{y}{9}\right) - 2\left(\sqrt{1-\frac{y}{9}}\right)\cdot\left(1-\frac{y}{9}\right) + \left(1-\frac{y}{9}\right)^2\right] dy$$

$$= \int_0^9 \left[\left(1-\frac{y}{9}\right) - 2\left(1-\frac{y}{9}\right)^{3/2} + \left(1-\frac{y}{9}\right)^2\right] dy$$

Can calculate separately:

$$\underbrace{\int_0^9 \left(1-\frac{y}{9}\right) dy}_{\left(y - \frac{1}{9}\frac{y^2}{2}\right)_0^9 = 9/2} - 2\int_0^9 \left[\left(1-\frac{y}{9}\right)^{3/2} \cancel{+} \left(1-\frac{y}{9}\right)^2\right] dy$$

Substitution would help here:

$$1 - \frac{y}{9} = u \Rightarrow \boxed{-\frac{1}{9}dy = du}$$

$y = 0 \Rightarrow u = 1$

$y = 9 \Rightarrow u = 0$

$$-2\cdot\left[\int_1^0 \left(u^{3/2} \cancel{+} u^2\right)(-9)\, du\right.$$

$$= -18\cdot\int_0^1 \left(u^{3/2} \cancel{+} u^2\right) du = \cancel{+}18\cdot\left[\frac{u^{5/2}}{5/2} \cancel{+} \frac{u^3}{3}\right]_0^1 = -\frac{6}{5}$$

So, the total volume is:

$$V = 9/2 - 6/5 = \frac{33}{10}.$$

(5) If cross-sections are triangles, then they are sitting their foot parallel to x or y. (x is harder, 2 integrals)

Lets do y: Check Purple line, that will be the foot of your triangle

$$V = \int_0^9 \underbrace{(\text{Area of triangle})}_{!} \cdot dy = \frac{1}{4} \int_0^9 \underset{?}{a^2}\, dy$$

$$A(\triangle) = \frac{ah}{2},$$

and it's equilateral $\Rightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{a^2}{4}$

so, $A = \left(\frac{a}{2}\right)\cdot\left(\frac{a}{2}\right) = \frac{a^2}{4}$

Find $\underline{\underline{a}}$ in terms of y

If $y = 9 - 9x^2$, then $9x^2 = 9 - y$

$\Rightarrow x^2 = 1 - \frac{y}{9} \Rightarrow x = \sqrt{1 - \frac{y}{9}}$,

(we again take the positive one, b/c we are on the right had side of the x axis!)

$y = 9 + 9x \Rightarrow x = \frac{y}{9} - 1$.

So, $a = \sqrt{1 - \frac{y}{9}} - \left(\frac{y}{9} - 1\right)$

$$\Rightarrow V = \int_0^9 \left[\sqrt{1 - \frac{y}{9}} - \left(\frac{y}{9} - 1\right)\right]^2 dy$$

As before, Square it first, then integrate & Evaluate!