Review for EXAM 1

We mainly covered Applications of integration:

1) Area between Curves
2) Volume
3) Arc length
4) Surface Area of a surface of revolution
5) Mass and Work

I - AREA:

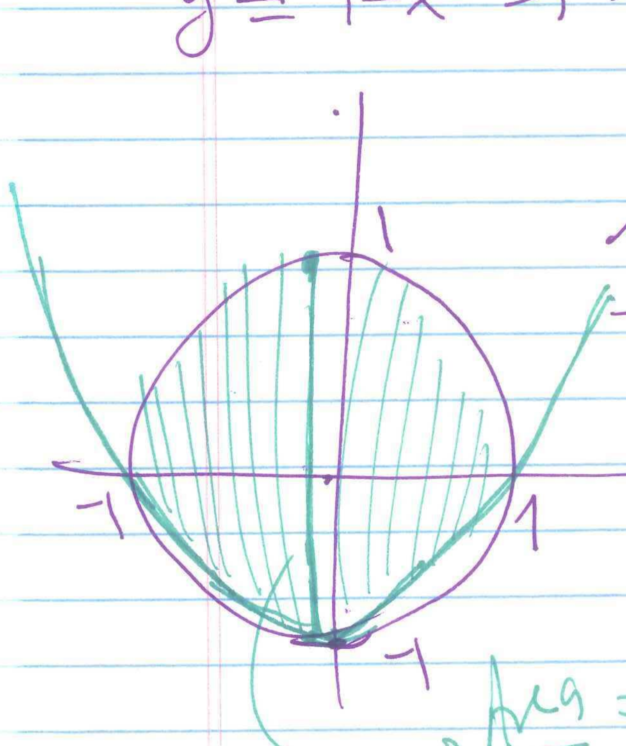
Ex: Graph the equations, shade the area of region btw curves. Determine its area.

$y = \sqrt{1-x^2}$, $\quad y = x^2 - 1$

$\swarrow$ square

$y^2 = 1 - x^2 \Rightarrow x^2 + y^2 = 1$ Circle with center $(0,0)$ radius $= 1$

$y = \sqrt{1-x^2}$ is its upper part.



Intersection pts: $\sqrt{1-x^2} = x^2 - 1 \Rightarrow 1 - x^2 = x^4 - 2x^2 + 1$ (square)

$\Rightarrow x^4 - x^2 = 0$

$\Rightarrow x^2(x^2 - 1) = 0$

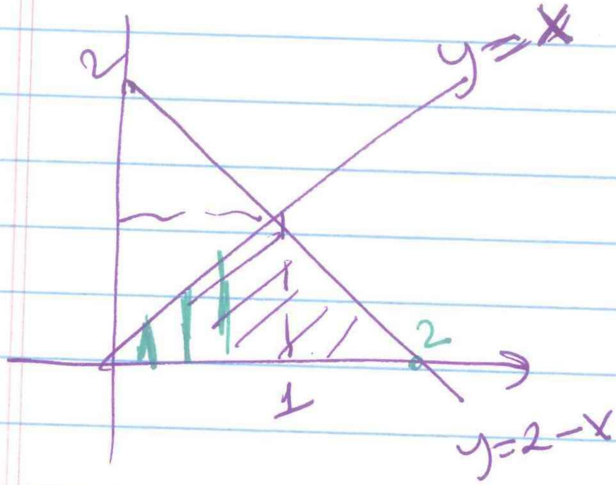
$x = 0, \; x = \pm 1$

$y = x^2 - 1$ is parabola at $y = -1$

$$\text{Area} = \int_{-1}^{1} \left[\left(\sqrt{1-x^2}\right) - \left(x^2 - 1\right) \right] dx$$ (We won't integrate this. Now)

(II) VOLUME, & AREA. (2)

Region bdd by graphs of $y=x$, $y=2-x$ and x axis:



$x=2-x \Rightarrow 2x=2$
$\Rightarrow \boxed{x=1}$

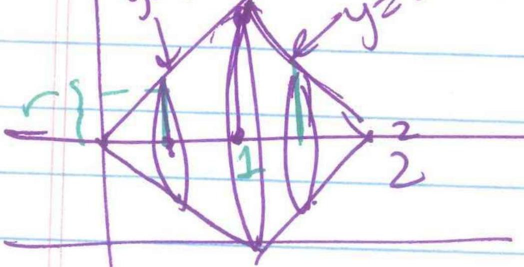
Area can be done 2 ways: (1) along x:

$$A=\int_0^1 x\,dx+\int_1^2 (2-x)\,dx$$

or: $A=\int_0^1 [(2-y)-y]\,dy=\int_0^1 (2-2y)\,dy$ ($y=2-x \Rightarrow x=2-y$)

$=\left(2y-y^2\right)\Big|_0^1 = 2-1=1$

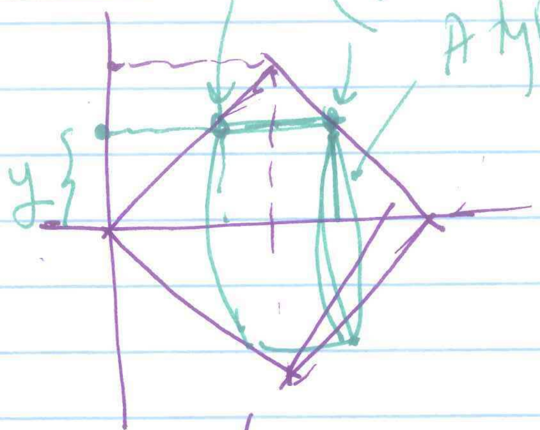
(b) REVOLVE it AROUND X axis:



VOLUME? with DISK METHOD:

$$V=\pi\cdot\int_0^1 x^2\,dx+\int_1^2 (2-x)^2\,dx$$

With Shell:

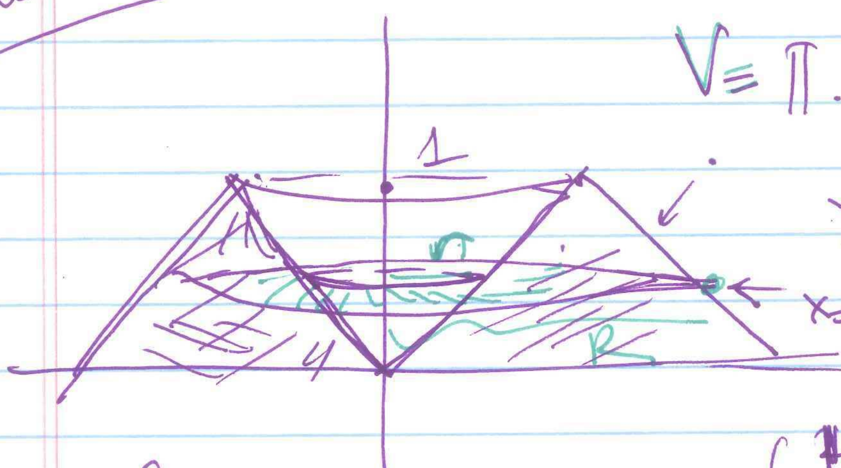


$$2\pi \int_{y=0}^{y=1} \overset{r}{y}\,\underbrace{\left[(2-y)-y\right]}_{(2-2y)}\,dy$$

$$=2\pi \int_0^1 (2y-2y^2)\,dy = 2\pi\left[y^2-\frac{2y^3}{3}\right]_0^1$$

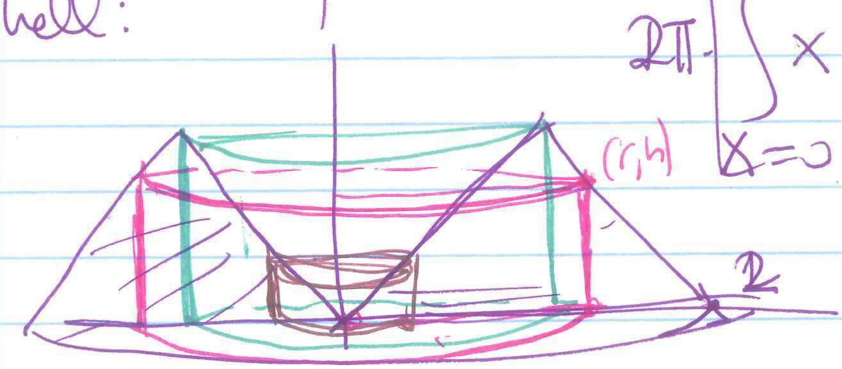
$$=2\pi\left[1-\frac{2}{3}\right] = 2\pi/3$$

Revolve around y:



$$V = \pi \int_{y=0}^{y=1} \left[(2-y)^2-y^2\right] dy$$

With shell:



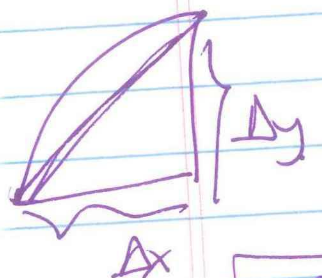
$$2\pi\left[\int_{x=0}^{1} x(\;\;x)\,dx + \int_{x=1}^{2} x\cdot(2-x)\,dx\right]$$

III Arc-lengths

Find the length of the segment of the curve

$y=\frac{3}{4}x^4+\frac{1}{24x^2}$ from $x=1$ to $x=2$

$$AL=\int_{x=1}^{2}\sqrt{1+\left(3x^3-\frac{1}{12x^3}\right)^2}\,dx$$



$$\sqrt{(\Delta x)^2+(\Delta y)^2}$$

$$=\sqrt{(\Delta x)^2\left[1+\left(\frac{\Delta y}{\Delta x}\right)^2\right]}$$

etc.

$$\boxed{\begin{aligned} y' &= \frac{3}{4}4x^3+\frac{1}{24}\cdot x^{-3}(-2) \\ &= 3x^3-\frac{1}{12x^3}\end{aligned}}$$

So, $AL=\int_1^2\sqrt{1+9x^6+\frac{1}{144x^6}-\frac{1}{2}}\,dx$

$$=\int_1^2\sqrt{\left(3x^3+\frac{1}{12x^3}\right)^2}\,dx=\int_1^2\left(3x^3+\frac{1}{12}x^{-3}\right)dx$$

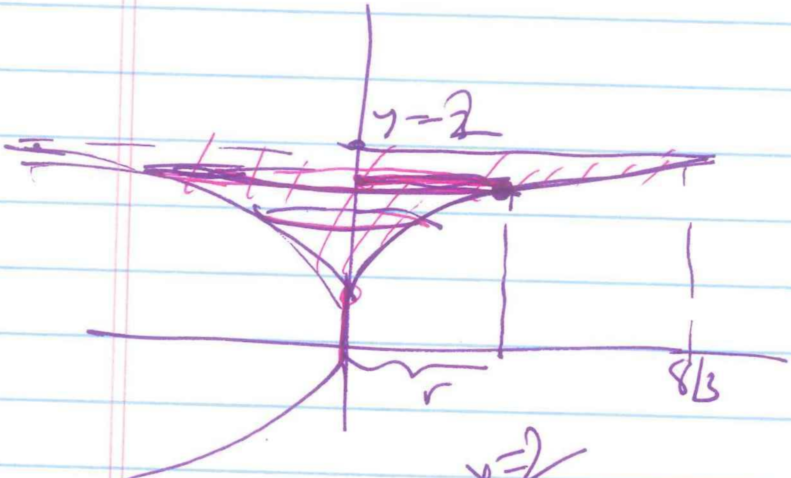
$$=\left.\left(\frac{3x^4}{4}+\frac{x^{-2}}{(12)(-2)}\right)\right|_1^2=\left(12-\frac{1}{96}\right)-\left(\frac{3}{4}-\frac{1}{24}\right)=\frac{1034}{96}$$

IV SURFACE AREA

Let $f(x) = y = \sqrt[3]{3x}$. Consider the portion of the curve $0 \le y \le 2$

Find the surface area of the surface generated by revolving graph of f(x) around the y-axis

$y = \sqrt[3]{3x} \Rightarrow x = g(y) = \frac{1}{3} y^3$



$y = 2 \Rightarrow \frac{1}{3} \cdot (2)^3 = \frac{8}{3}$

$g(y) = \frac{1}{3} y^3$

$\Rightarrow g'(y) = y^2$

$$SA = 2\pi \int_{y=0}^{y=2} \left(\frac{1}{3}\right) y^3 \sqrt{1 + (y^2)^2}\, dy$$

$1 + y^4 = u$
$\Downarrow$
$4y^3\, dy = du$

$$\int y^3 \cdot \sqrt{1+y^4}\, dy = \frac{1}{4} \int \sqrt{u}\, du = \frac{1}{4} \frac{u^{3/2}}{3/2} = \frac{1}{6} u^{3/2} + C$$

$$SA = \frac{\pi}{9} \left(1 + y^4\right)^{3/2} \Big|_0^2 = \frac{\pi}{9} \left[17^{3/2} - 1 \right] = \frac{\pi}{9} \left[17\sqrt{17} - 1 \right]$$

6

V WORK & MASS:

CONSTANT FORCE

- Work done $\boxed{\text{lifting a 50 lb}}$ box onto a truck, which is 3 ft off the ground, is:

$$\underline{\underline{(50)\times 3 = 150 \text{ ft lb}}}$$

- To lift a 20kg child from the floor to a height of 2m?

CONSTANT FORCE

$$(20)\cdot(9,8)\cdot 2 = 392\ J$$

$(1\ kg = 9.8\ N)$

non-constant force

This is function

- Work done for a force $F = \frac{12}{x^2}$ from $x=1$ to $x=2$

$$W = \int_1^2 \frac{12}{x^2}dx = 12\int_1^2 x^{-2}dx = 12\frac{x^{-1}}{-1}\Big|_1^2$$

$$= -12\left[\frac{1}{2} - 1\right] = 6\ J.$$