BASIC INTEGRATION FORMULAS

(1) Power Rule

(a)

$$\int_1^4 \sqrt{t}(1+t)\,dt = \int_1^4 \left(\sqrt{t} + t\sqrt{t}\right) dt = \int \left(t^{1/2} + t^{3/2}\right) dt$$

$$= \left(\frac{t^{3/2}}{3/2} + \frac{t^{5/2}}{5/2}\right)\Bigg|_1^4 = \frac{256}{15}$$

(b)

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = \int \left(x^{1/2} - x^{-1/2}\right) dx = \frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2}$$

$$= \frac{2}{3}x^{3/2} - 2x^{1/2} + C$$

$$\int \left(e^{2x} - \frac{1}{2}e^{x/2}\right) dx$$

$$= \frac{e^{2x}}{2} - \frac{1}{2}\cdot\frac{e^{x/2}}{1/2} + C$$

(2) SUBSTITUTION:

Ex. $I = \int 6x(3x^2+4)^4\,dx$ (Hint: Polynomial & Polynomial with one less degree (can't))

$6x$: Degree 1 polynomial; $3x^2+4$: Degree 2 polynomial

So, substitute: $u = 3x^2+4$

Then: $du = 6x\,dx$

$\Rightarrow I = \int u^4\,du = \frac{u^5}{5} + C = \frac{(3x^2+4)^5}{5}$

Ex 2 $\int \frac{\sin t}{\cos^3 t}\,dt$

Hint: Sin & cos. are derivatives of each other.

Take Deriv. of both

Let $u = \cos t \Rightarrow du = \sin t\,dt$

(If you let $u = \cos^3 t$

Then $du = 3\cos^2 t \sin t\,dt$

You don't have this

With our substitution

$$I = \int \frac{du}{u^3} = \int u^{-3}\,du = \frac{u^{-2}}{-2} + C$$

$$= \frac{-1}{2\cos^2 t} + C$$

Ex 3: $I = \int \frac{x}{\sqrt{x-1}}\,dx$?

$x - 1 = u \Rightarrow dx = du$

(Then: $x = u + 1$)

$$I = \int \frac{(u+1)}{\sqrt{u}}\,du = \int \left(\sqrt{u} + \frac{1}{\sqrt{u}}\right) du$$

$$= \frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} + C = 2\frac{(x-1)^{3/2}}{3} + 2(x-1)^{1/2} + C$$

(3) EXPONENTIAL & LOGARITHMIC FUNCTIONS:

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

(Recall: $(a^x)' = (a^x)\cdot \ln a$)

Ex $\int x^2 \cdot e^{-2x^3} dx = -\frac{1}{6}\int e^u du = -\frac{1}{6}e^u + C = -\frac{1}{6}e^{-2x^3} + C$

(Substitute: $u = -2x^3$. Then $du = -6x^2 dx$)

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \ln x \, dx = x\ln x - x + C$$

later: Substitute; IBP

$$\int \log_a x \, dx = \frac{x}{\ln a}(\ln x - 1) + C$$

EX: $\int \frac{1}{x+2} dx = \ln|x+2| + C$

Ex: $\int_0^{\pi/2} \frac{\sin x}{1+\cos x}\,dx$

Substitute: $u = 1+\cos x$

$du = -\sin x\,dx$

$x = 0 \Rightarrow u = 1+\cos 0 = 2$

$x = \pi/2 \Rightarrow u = 1+\cos\frac{\pi}{2} =$

$$= \int_2^1 \frac{du}{u} = \int_1^2 \frac{du}{u} = \ln|u|\Big|_1^2 = \ln 2$$