2/6/20

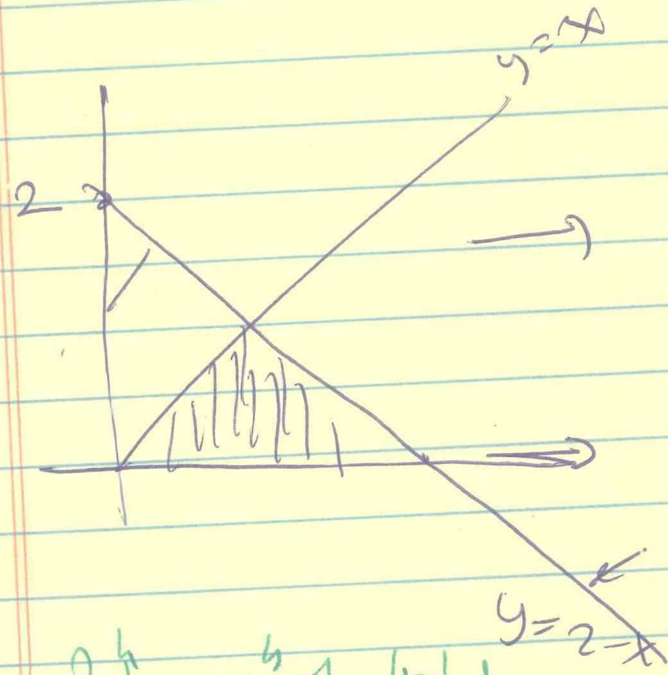
$\boxed{\text{EX}}$ SELECTING THE BEST METHOD:

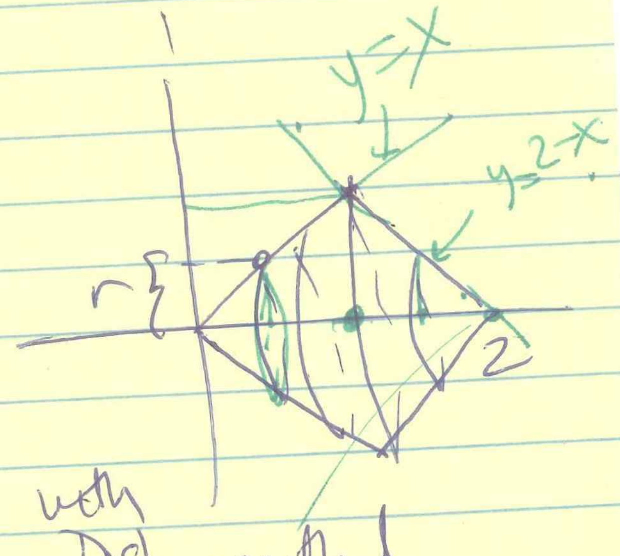
~~Sketch~~ Revolve around x axis the following:

(a) Region bdd by graphs of $y=x$, $y=2-x$, and the x axis

(b) Region bdd by graphs of $y=4x-x^2$ and x axis

(a)





with Disk method

There are 2 types of disks

$$V=\pi\int_0^1 x^2\,dx+\pi\int_1^2 (2-x)^2\,dx$$

Two integrals.

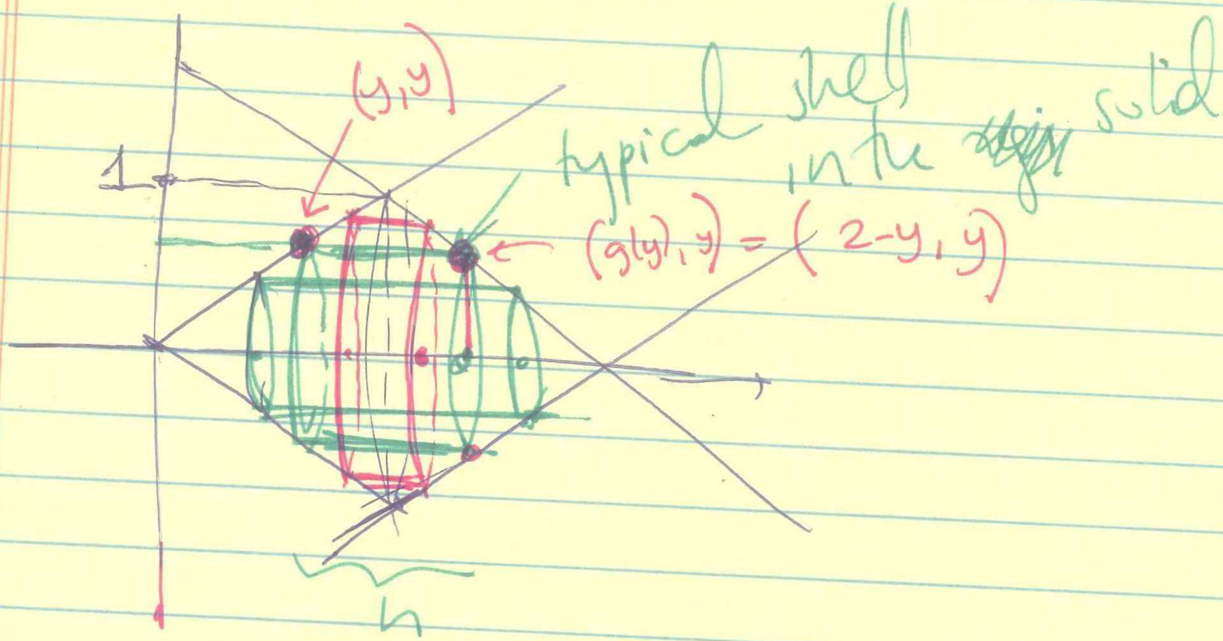
intersection

$x=2-x$

$2x=2$

$\Rightarrow x=1$

If we do this with shell method:



So, $h = (2-y) - y$.

$$V = 2\pi \cdot \int_0^1 2\pi y \cdot \left[(2-y) - y\right] dy = \int_0^1 2\pi y\,(2-2y)\, dy$$

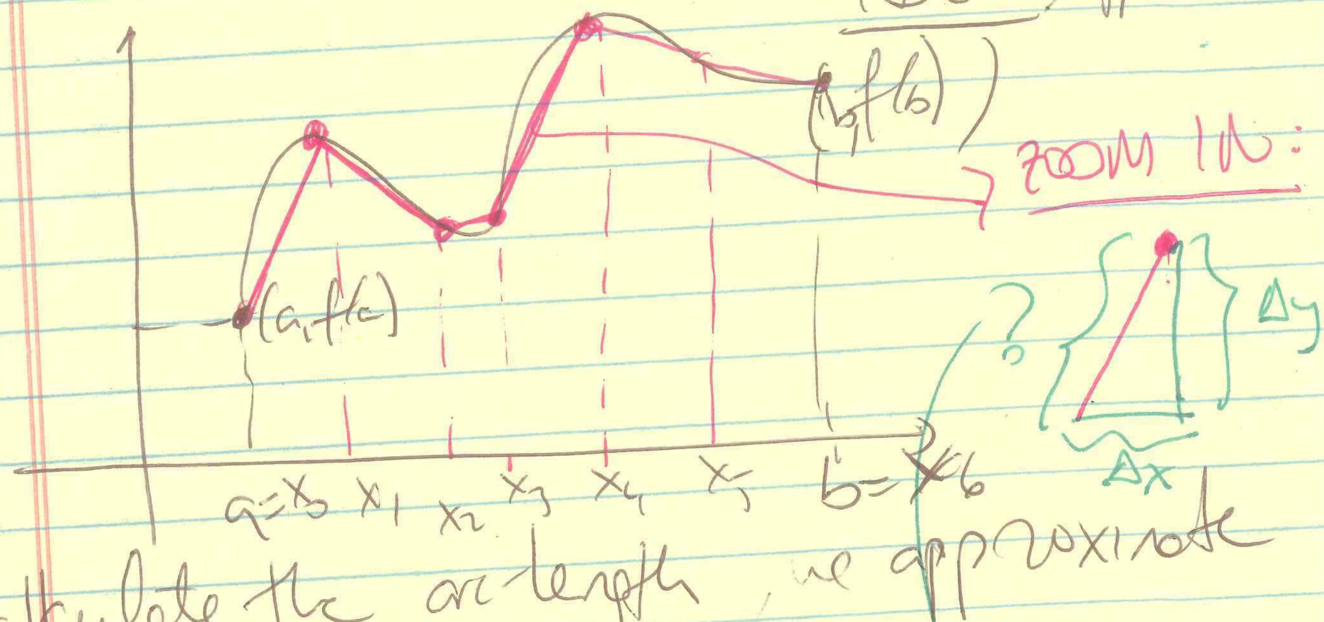
ARC LENGTH & SURFACE AREA:

Arc length = Distance you would travel if you were walking along the path of the curve.

- Arclength of a curve given by $y = f(x)$

Want: Integrable function, diff'ble and derivative f' continuous

IDEA Approximate



To calculate the arc length, we approximate the curve with lines.

$$? = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Write this as: $\Delta x \cdot \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$.

When we add up these lengths,

$$\text{Arc length} \approx \sum_{i=1}^{n} \sqrt{1+\left(\frac{\Delta y_i}{\Delta x}\right)^2}\,\Delta x \qquad \left(\begin{array}{c}\Delta x \text{ the same,}\\ \Delta y \text{ changes}\end{array}\right)$$

This is derivative of f at some pt $x_i^* \in [x_{i-1}, x_i]$

So,

$$\mathcal{L} = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

$\boxed{EX}$ Let $f(x) = 2x^{3/2}$. What is the arc length of graph of $f(x)$ over the interval $[0,1]$? Round to 3 decimal places

$$\mathcal{L} = \int_0^1 \sqrt{1+(f'(x))^2}\,dx = \int_0^1 \ldots\,dx$$

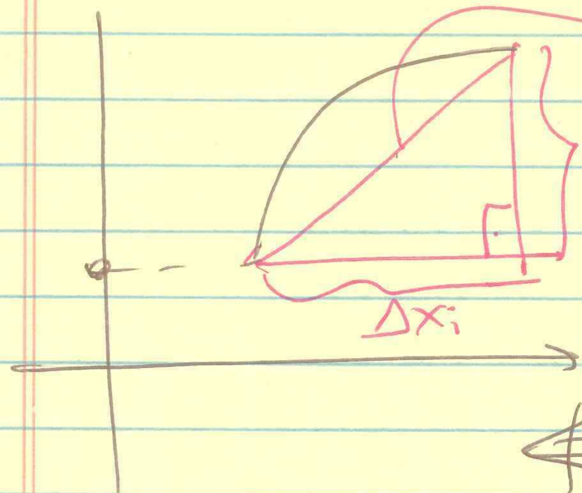
$$f'(x) = 3x^{1/2} \qquad = \frac{1}{9}\int_0^1 \sqrt{1+9x}\,dx = \frac{1}{9}\int_1^{10} \sqrt{u}\,du = \frac{2}{3}u^{3/2}$$

$$= \frac{1}{9}\cdot\frac{2}{3}\left(10^{3/2}-1\right) = \frac{2}{27}\left(10\sqrt{10}-1\right)$$

$$\boxed{\begin{array}{l} 1+9x = u \Rightarrow 9\,dx = du \\ x=0 \Rightarrow u=1,\ x=1 \Rightarrow u=10 \end{array}}$$

Arclength of $x = g(y)$

Now, Δy is fixed, Δx changing, so



This segment has length:

$$\sqrt{(\Delta x_i)^2 + (\Delta y)^2}$$

$$= \Delta y \sqrt{\left(\frac{\Delta x_i}{\Delta y}\right)^2 + 1}$$

$\frac{\Delta x_i}{\Delta y}$ → derivative of $g(y)$ wrt y

$$\text{Arc length} = \int_c^d \sqrt{1+[g'(y)]^2}\,dy.$$

$\boxed{\text{Ex}}$ Let $g(y) = 3y^3$, find arclength of $g(y)$ over the interval $[1,2]$

$g'(y) = 9y^2$.

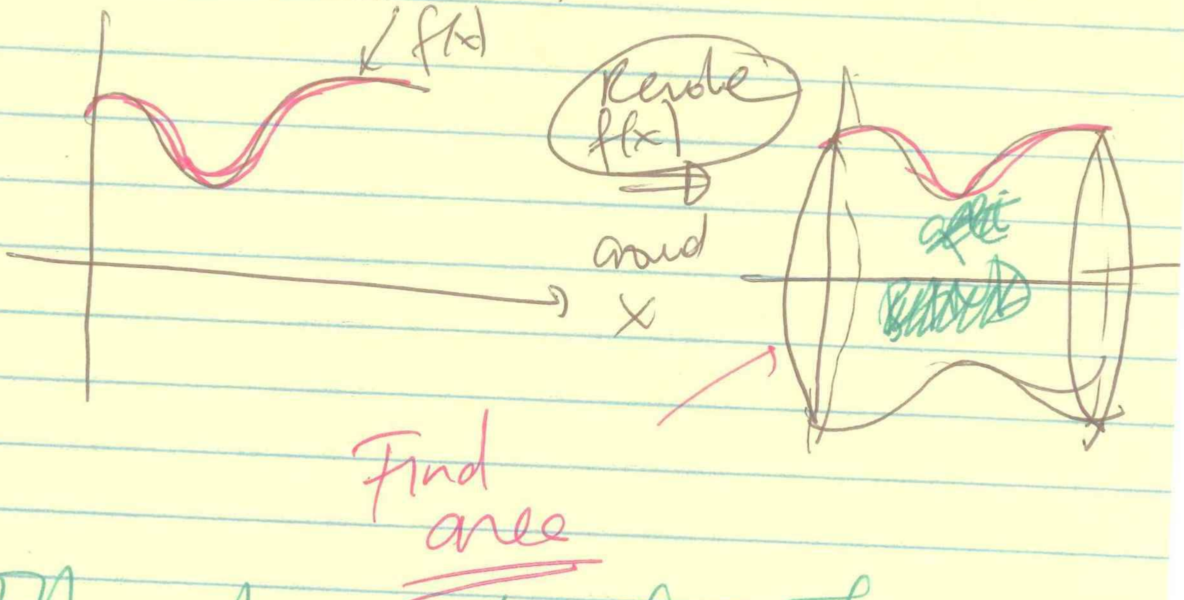
So,

$$L = \int_1^2 \sqrt{1+(9y^2)^2} \approx$$

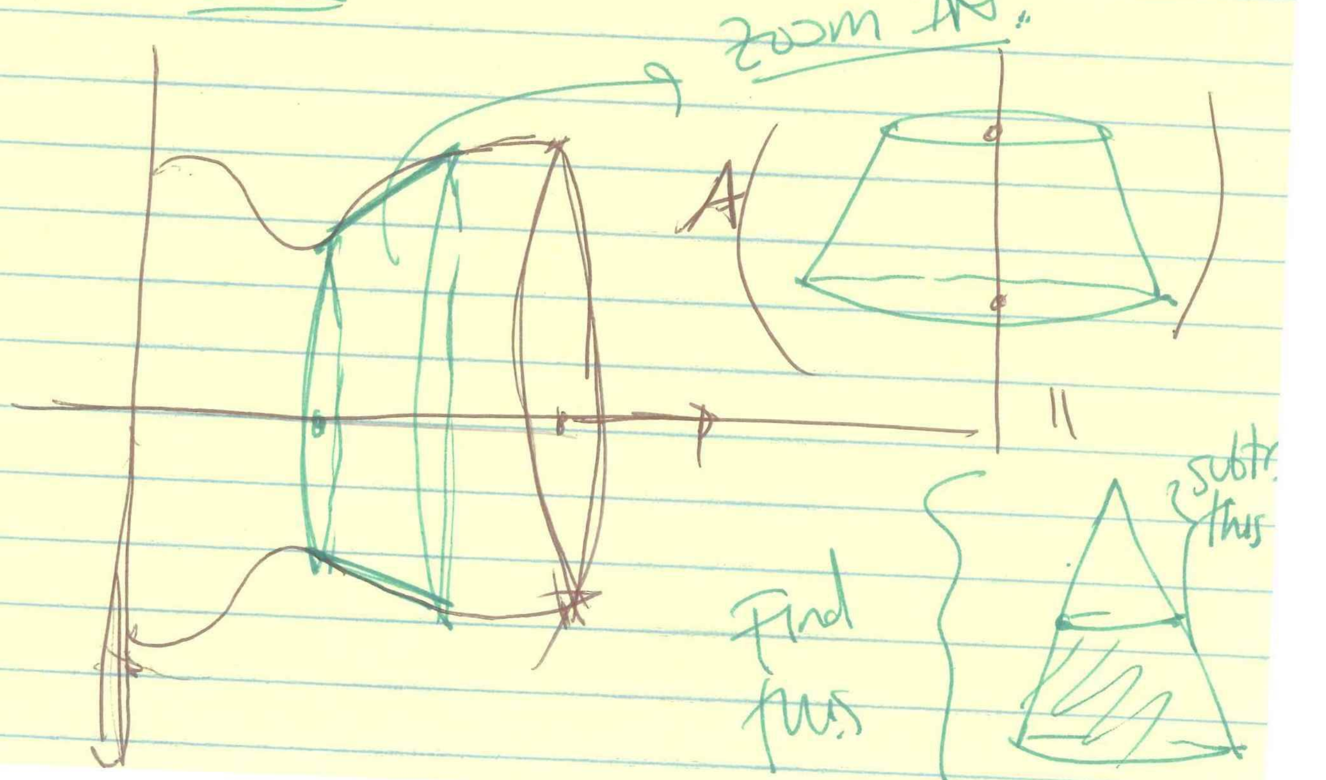
USING CALCULATOR

SURFACE AREA of a surface of revolution

Today:



IDEA: Approximate f(x). This lets us divide the region into bands:

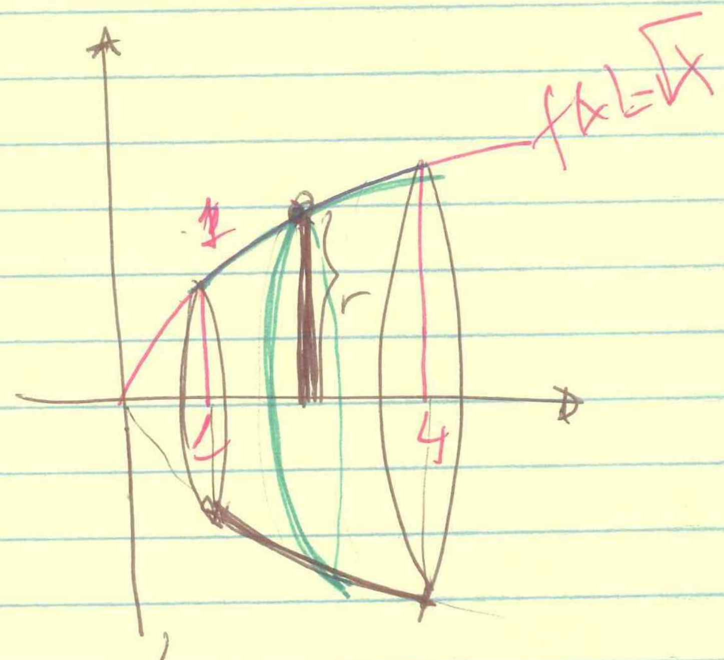


We'll do:

$$S = \int_a^b \underbrace{2\pi r}\cdot(\text{Arclength}) = \int_a^b$$

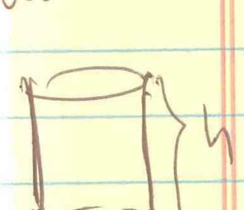
$$= \int_a^b 2\pi\cdot f(x)\left[\sqrt{1+(f'(x))^2}\right]dx$$

Ex:



Let $f(x)=\sqrt{x}$ over the interval $[1,4]$. Find the area of the surface obtained by revolving the graph of f along x axis

Compare with



$$S.A = \int_1^4 2\pi\cdot\sqrt{x}\cdot\left[\sqrt{1+\left(\frac{1}{4x}\right)}\right]dx = 2\pi\int_1^4 \cancel{\sqrt{x}}\cdot\frac{\sqrt{1+4x}}{2\cancel{\sqrt{x}}}\,dx$$

$$f(x)=\sqrt{x} \Rightarrow f'(x)=\frac{1}{2\sqrt{x}}$$

$1+4x=u \Rightarrow 4dx=du$, $x=1 \Rightarrow u=5$
$x=4 \Rightarrow u=17$

$$SA = \frac{\pi}{4}\int_5^{17} u^{1/2}\,du = \frac{\pi}{4}\left[\frac{2}{3}u^{3/2}\right]_5^{17} =$$