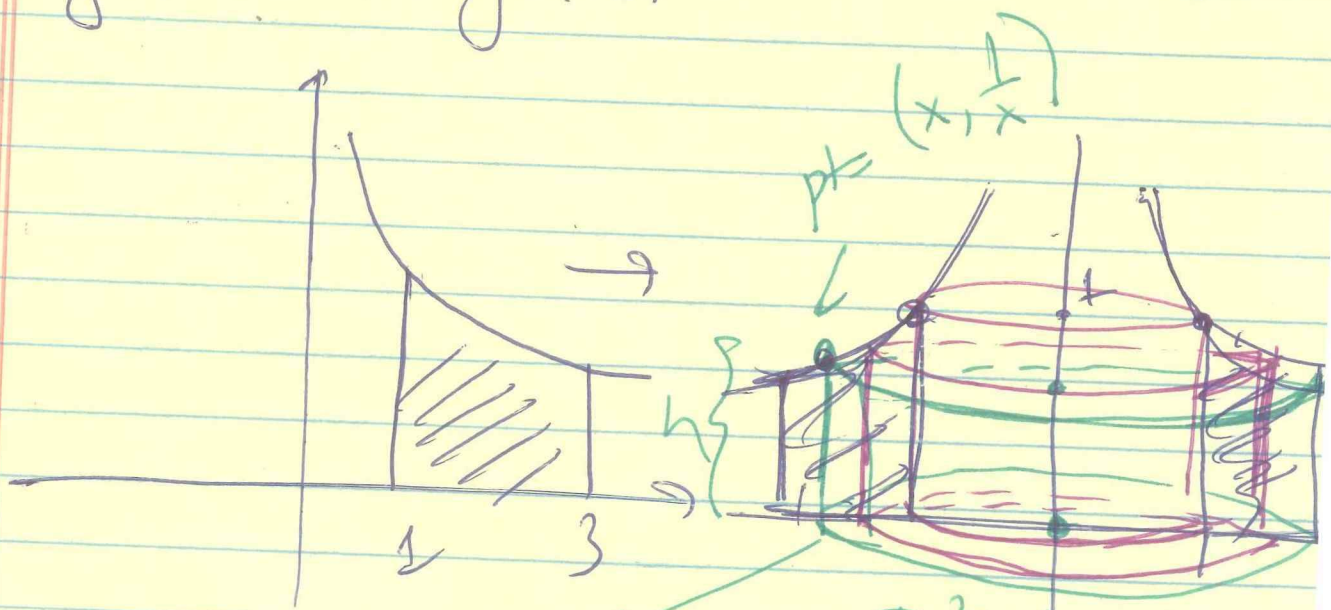
Week 7 Feb 8 / Part 1

Q: Is there an easier way of finding the volume?

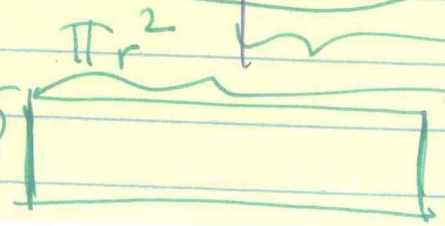
CYLINDRICAL SHELLS:

IDEA: Scoop out of the solid! (Instead of slicing.)
(CYLINDRICAL shells)

Ex R: Bdd above by $f(x) = \frac{1}{x}$, below x axis over $[1,3]$
Find the volume of the solid formed by revolving R around y axis:



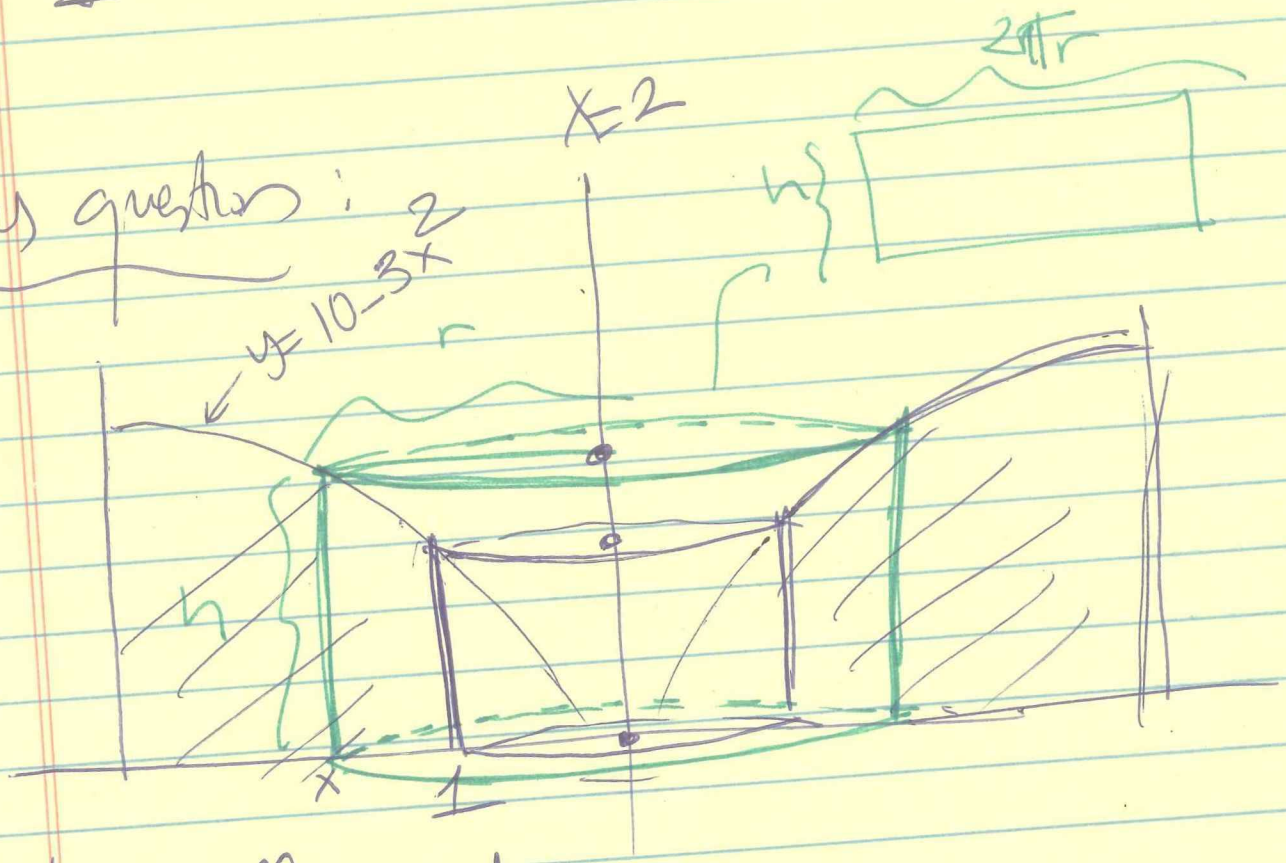
Cut the shell to rectangle:



$$V = 2\pi \int_{1}^{3} (x) \cdot \left(\frac{1}{x}\right) dx = 2\pi \cdot x \Big|_1^3 = 4\pi$$

(r = x, f(x) = 1/x)

Previous question:



With shell method

$$V = 2\pi \int_0^1 \left[h \cdot (10 - 3x^2) \right] \underbrace{(2 - x)}_{r} \, dx$$

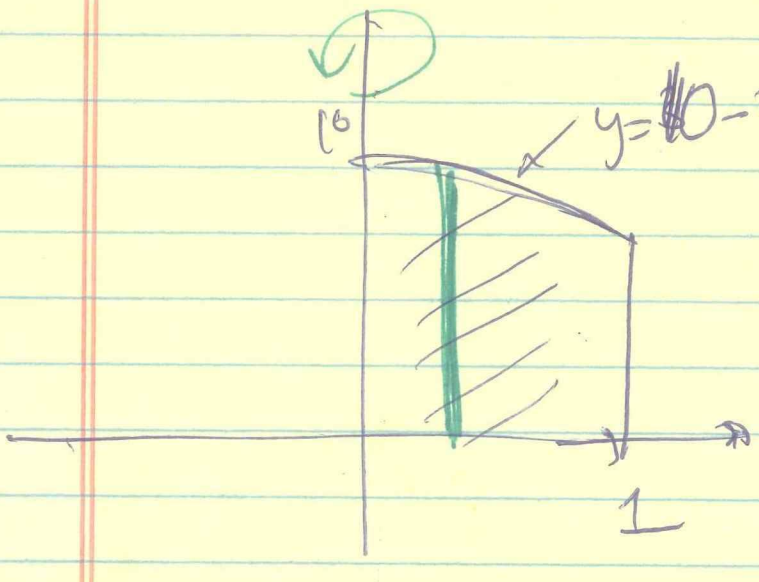
$$= 2\pi \cdot \int_0^1 (20 - 10x - 6x^2 + 3x^3) \, dx = \frac{89\pi}{2}$$

TIP: Use this method when "vertical" rectangles are parallel to axis of rotation!

Ex

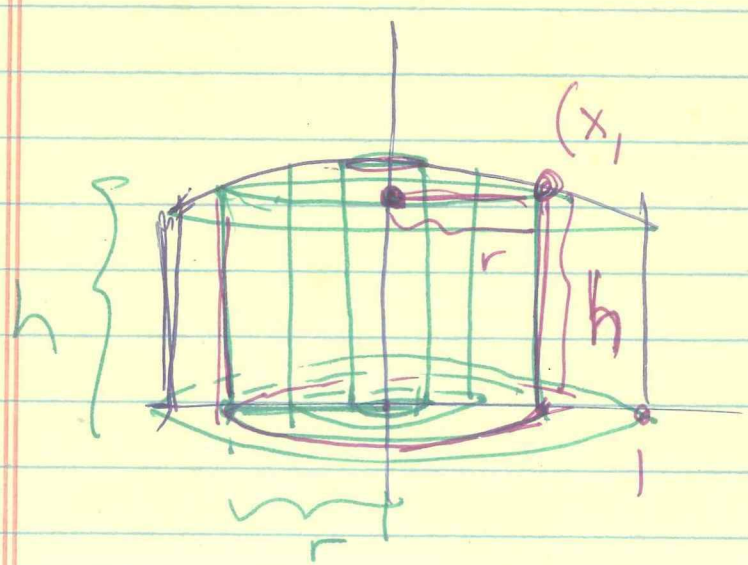
Problem 6 again, $y = 10 - 3x^2$, $x=0$, $x=1$

$x=0$ axis



I'll pick my rectangles vertical b/c there is only one type if I do so (otherwise there will be 2 types)

Now, my rectangle (which is vertical) parallel to the axis of rotation (y axis) So I use SHELL method.

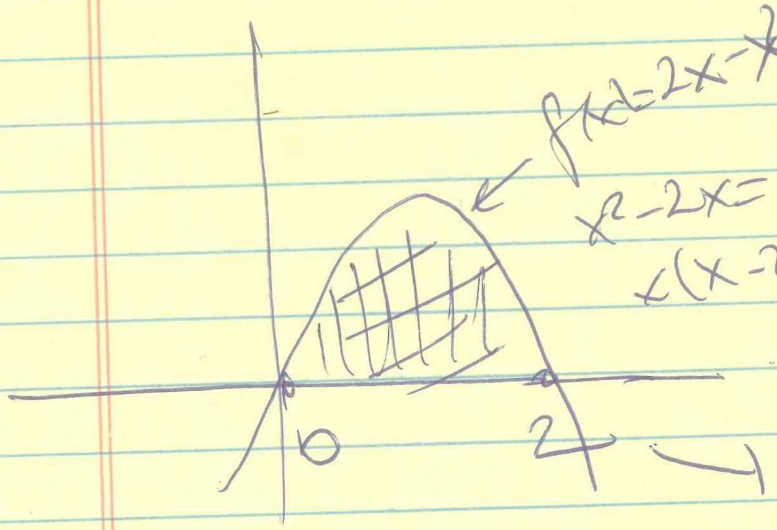


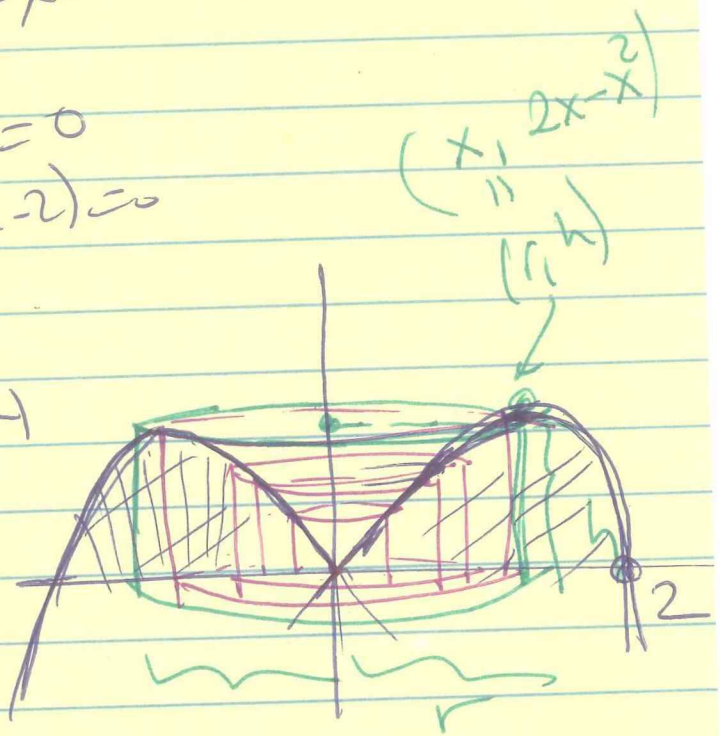
$$V = 2\pi \int_{x=0}^{x=1} x \cdot (10 - 3x^2)\, dx$$

$$= \frac{17\pi}{2}$$

Ex: R: bdd by $f(x) = 2x - x^2$, below x axis, on $[0,2]$

Revolve around y axis, find volume

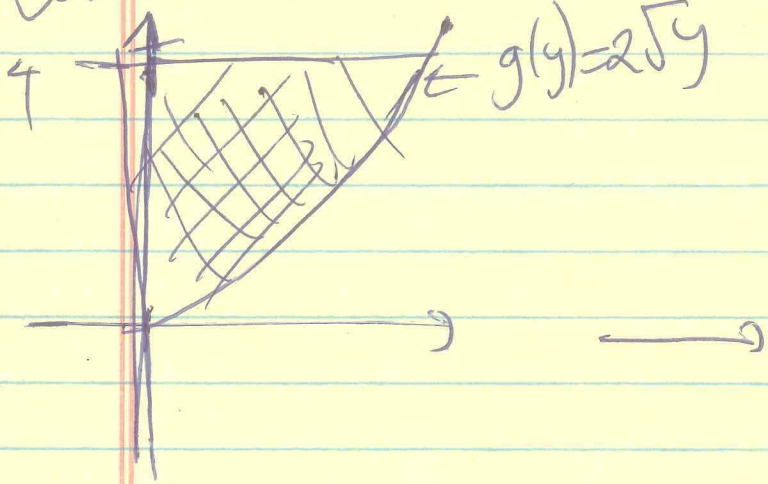


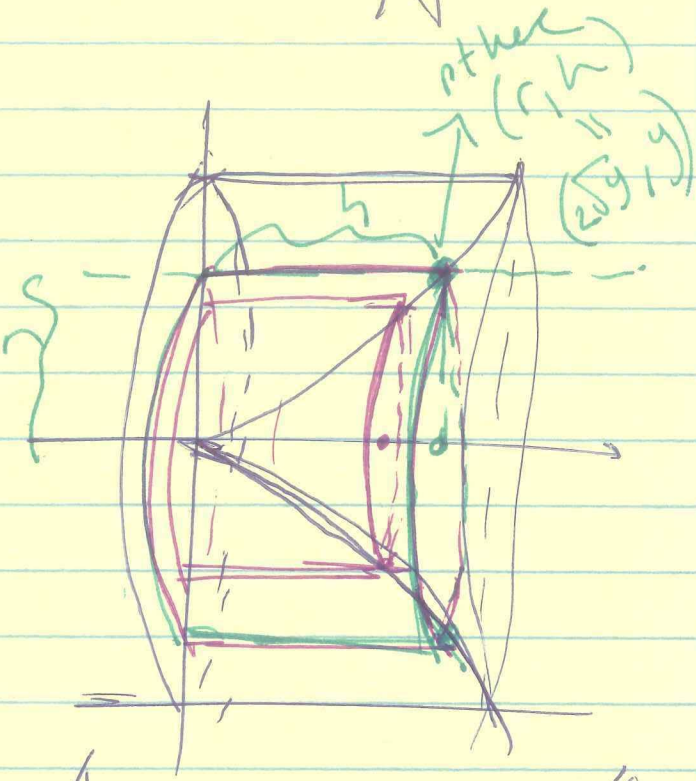


$$V = 2\pi \int_0^2 (x)\cdot(2x - x^2)\,dx = 2\pi \cdot \int_0^2 (2x^2 - x^3)\,dx$$

$$= 2\pi \cdot \left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_0^2 = 2\pi \cdot \left[\frac{16}{3} - \frac{16}{4}\right] = \frac{8\pi}{3}$$

Ex: 8: Region bdd by $g(y) = 2\sqrt{y}$, y axis for $y \in [0, 4]$. Revolve around x axis, find volume.





$$V = 2\pi \int_{y=0}^{y=4} (2\sqrt{y}) \cdot y \, dy = 2\pi \int_0^4 2y^{3/2} \, dy = 4\pi \frac{y^{5/2}}{5/2} \Big|_0^4$$

$$= \frac{8\pi}{5} \cdot (2^5) = \frac{256\pi}{5}$$