EX $I = \int \frac{\ln x}{x^3}\,dx = \int x^{-3} \ln x\,dx$

If you pick $u = x^{-3}$, it's harder to integrate $\ln x$.

So; we pick $u = \ln x \Rightarrow du = \frac{1}{x}dx$

$x^{-3}dx = dv \Longrightarrow \int x^{-3}dx = \int dv$

Integrate $\Rightarrow \frac{x^{-2}}{-2} = v$

$$\Rightarrow I = \underbrace{\frac{-1}{x^2}}_{v} \underbrace{\ln x}_{u} + \int \frac{1}{x^2} \underbrace{\frac{1}{x}\,dx}_{du}$$

$$= -\frac{1}{x^2}\ln x + \frac{x^{-2}}{-2} + C$$

$$= \boxed{-\frac{1}{x^2}\ln x - \frac{1}{2x^2} + C}$$

EX $\int x^2 e^{3x}\,dx$. (IBP twice)

$u = x^2 \Rightarrow du = 2x\,dx$

$e^{3x}dx = dv \Rightarrow \boxed{\frac{e^{3x}}{3} = v}$

$I = \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int e^{3x} \cdot x\,dx$ IBP here again.

$u = x \Rightarrow du = dx$

$e^{3x}dx = dv \Rightarrow \frac{e^{3x}}{3} = v.$

$$\Rightarrow I = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x}dx\right]$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$

Ex

SOMETIMES "LIATE" DOESN'T quite WORK!

Ex: $\int t^3 e^{t^2} dt$

If $t^3 = u$, then $3t^2dt = du$.

then we can't integrate $\int e^{t^2}dt$!

So, we will think: $\int t^3 e^{t^2} dt = \int \underbrace{t^2}_{u} \cdot \underbrace{te^{t^2}dt}_{dv}$

$t^2 = u \Rightarrow 2t\,dt = du.$

$\int te^{t^2}dt = \int dv \Rightarrow \boxed{\frac{1}{2}e^{t^2} = v}$

Substitution

$t^2 = w \Rightarrow 2t\,dt = dw$

$\boxed{EX}$ $\int \sin(\ln x)\, dx$

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$\sin(\ln x) = u \Rightarrow \frac{1}{x}\cos(\ln x)\, dx = du.$

$dx = dv \Longrightarrow x = v$

Integrate

$\Rightarrow I = x \cdot \sin(\ln x) - \int x \cdot \frac{1}{x} \cos(\ln x)\, dx$

Now again: $\cos(\ln x) = u$ $\Downarrow$ $-\frac{1}{x}\sin(\ln x)\, dx = du.$ | $dx = dv$ $\Downarrow$ $x = v.$

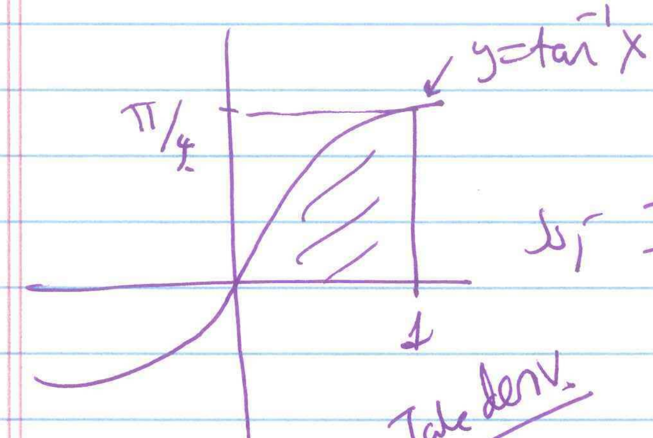
$\Rightarrow I = x\sin(\ln x) - \left[x \cdot \cos(\ln x) + \underbrace{\int \frac{1}{x} \cdot x \cdot \sin(\ln x)\, dx}_{I.}\right]$

$\Rightarrow 2I = x \cdot \sin(\ln x) - x \cdot \cos(\ln x)$

$\Rightarrow \boxed{I = \frac{1}{2} x\sin(\ln x) - \frac{1}{2} x\cos(\ln x) + C}$

EX Definite Integrals:

Find the area of the region bdd above by the graph of $y = \tan^{-1}x$ and below by the x-axis over the interval $[0,1]$.



so, $I = \int_0^1 (\tan^{-1}x)\,dx$.

Take deriv.

Let $u = \tan^{-1}x \Rightarrow \boxed{du = \frac{1}{1+x^2}dx}$. $\quad dx = dv \Rightarrow x = v$ Integrate

If $u = \tan^{-1}x$

$\Downarrow$

$x = \tan u$

$\Downarrow$ Deriv. wrt x

$1 = (1+\tan^2 u)\cdot\frac{du}{dx}$

$\Downarrow$

$du = \frac{1}{1+\tan^2 u}dx$ ← put this in terms of x

$\rightarrow \boxed{du = \frac{1}{1+x^2}dx}$

$$I = \int_0^1 (\tan^{-1}x)\,dx = x\tan^{-1}x\Big|_0^1 - \underbrace{\int_0^1 \frac{x}{1+x^2}dx}$$

Substitution here:
$1+x^2 = w$
$\rightarrow 2x\,dx = dw$

$$I = x\tan^{-1}x\Big|_0^1 - \frac{1}{2}\ln(1+x^2)\Big|_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}\left[\ln 2 - \underbrace{\ln 1}_{0}\right]$$

Find volume of revolution:

Ex Find the volume of the solid obtained by revolving the region bdd by the graph of $f(x) = e^{-x}$, the x axis, the y-axis, and the line $x = 3$, about the y axis

$x = 0 \Rightarrow y = e^{0} = 1$

with shell method:

$$2\pi \cdot \int_{x=0}^{3} x e^{-x} \, dx$$

IBP: $x = u \Rightarrow dx = du$

$e^{-x} dx = dv \Rightarrow -e^{-x} = v$

$$\frac{1}{2\pi} I = -xe^{-x} \Big|_0^3 + \int_0^3 e^{-x} dx$$

$$= -3e^{-3} + \left(\frac{e^{-x}}{-1} \right) \Big|_0^3 = -3e^{-3} - \left[e^{-3} - 1 \right] = -4e^{-3} + 1$$

$$\Rightarrow \boxed{I = 2\pi \cdot (-4e^{-3} + 1)}$$