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Work done to stretch or compress a spring:

EX Suppose it takes a force of 10 N in the neg. direction to compress a spring 0.2 m from the equilibrium position.

How much work is done to stretch the spring 0.5 m from the equilibrium position?

$$-10N = k(-0.2) \Rightarrow k = 50.$$

$$W = \int_a^b F(x)\,dx = \int_0^{0.5} 50 \cdot x\,dx = 6.25$$

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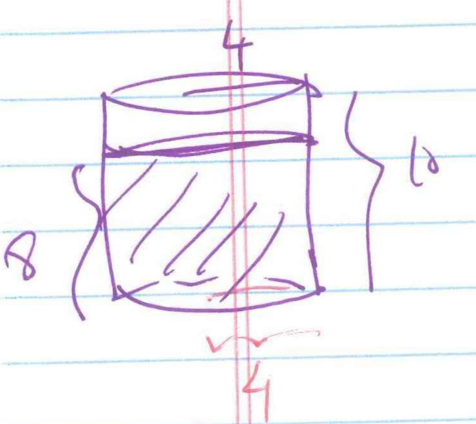
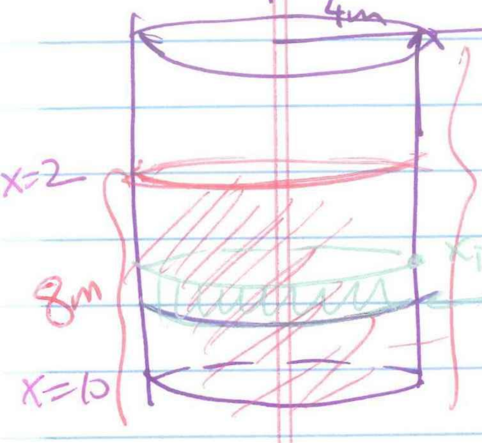
WORK DONE IN PUMPING

Assume we have a cylindrical tank. radius 4m, height = 10 m, filled to a depth of 8m



How much work does it take to pump all the water over the top edge of the tank?

We'll put the coordinate syst. as follows:



$x = 0$

Positive x direction

Partition this interval [2,10]

So, water extends from $x=2$ to $x=10$.

So partition [2,10] and look at the work required to lift each individual 'layer' of water.

So, for $i=1,..,n$. let

why are we doing this? B/c, it is easier to remove the water from the top, ~~so~~ force ~~is not~~ constant!! ~~so FORCE~~ is

Force required to lift the water = force required to overcome gravity = Weight of the water = Volume × Weight-density of water

Volume $= \pi r^2 \Delta x = \pi (4)^2 \Delta x$

weight density

$$F = (9800) \cdot 16\pi \cdot \Delta x.$$

distance the layer must be lifted $= x_i^*$

$$\boxed{W = (9800) \int_2^{10} \pi \cdot 16 x \, dx}$$

INTEGRATION BY PARTS:

$\int x\sin(x^2)\,dx$ v.s. $\int x\sin x\,dx$

Substitute $x^2 = u$. ?

We will do IBP:

Idea: Product rule

If $h(x) = f(x)g(x)$, $h'(x) = f'(x)g(x) + f(x)g'(x)$

$$\Rightarrow \int h'(x) = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$\Rightarrow \int f(x)g'(x)\,dx = \int h'(x) - \int f'(x)g(x)\,dx$$

$$\Rightarrow \boxed{\int fg' = fg - \int f'g}$$

We will make substitutions: $u = f(x)$, $v = g(x)$

$du = f'(x)\,dx$, $dv = g'(x)\,dx$

$$\Rightarrow \boxed{\int u\,dv = u\cdot v - \int v\,du}$$

IBP

Ex $\int \overbrace{x \sin x \, dx}^{u\,dv}$.

Choose something to differentiate, Rest to Integrate. (So pick a "u" and pick a "dv")

let $u = x$, $dv = \sin x \, dx$

Differentiate u $\Downarrow$ $\Downarrow$ Integrate

$\boxed{du = dx}$ $\int dv = \int \sin x \, dx$

$\boxed{v = -\cos x}$

So, $\int u\,dv = \int x \sin x\,dx = \underbrace{-x \cos x}_{uv} - \int \underbrace{(-\cos x)}_{v} \underbrace{dx}_{du}$

$= -x\cos x + \sin x + C$

Ex $I = \int x e^{2x}$, let $u = x \Rightarrow du = dx$

$\int e^{2x} dx = \int dv \Rightarrow \frac{e^{2x}}{2} = v$

$I = \frac{x \cdot e^{2x}}{2} - \frac{1}{2} \int e^{2x} dx = \frac{x e^{2x}}{2} - \frac{1}{2}\left[\frac{e^{2x}}{2}\right] + C$

$= \frac{x e^{2x}}{2} - \frac{1}{4} e^{2x} + C$

LIATE

L → Log

I → Inverse trig

A → Algebraic functions (like polynomials)

T → Trig functions

E → Exp. functions

ORDER to pick u. (Most of the time!!!)

Ex: $I = \int \ln x \, dx$ ← Logarithmic

(Recall: $\int \ln x \, dx \neq \frac{1}{x} + C$

So, let $u = \ln x$, $dv = dx$

Different. ⇓ ⇓ Integrate

$du = \frac{1}{x} dx$ $v = x$.

$$I = x \ln x - \int \underbrace{x}_{v} \cdot \underbrace{\frac{1}{x} dx}_{du} = x \ln x - x + C$$