2/10

MASS & DENSITY:

Consider a thin rod or wire,
Suppose it sits on x axis. (from $x=a$ to $x=b$)

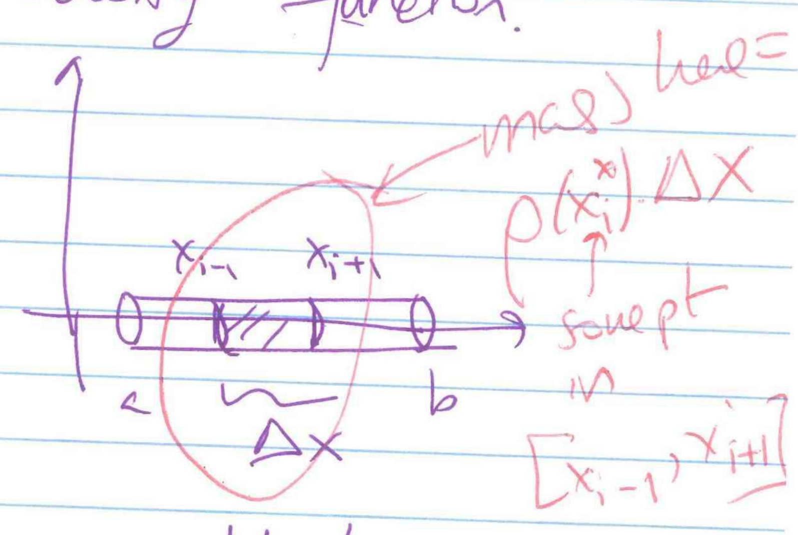
(ROD is THIN!)

If the rod has constant density ρ,

$$\text{Mass} = (b-a)\cdot\rho$$

But if it changes, then we will have $\rho(x)$, an integrable linear density function.

APPROXIMATION:



Calculate each small mass, add them up:

$$\Rightarrow \text{MASS} = \int_a^b \rho(x)\,dx$$

Ex: A thin rod is oriented on x axis over the interval $[\pi/2, \pi]$.

If density is given by $\rho(x) = \sin x$, what is the mass of the rod?

Sln: $m = \int_{\pi/2}^{\pi} \sin x \, dx = -\cos x \Big|_{\pi/2}^{\pi} = 1$

Ex. If $\rho(x) = 2x^2 + 3$, what is the mass of a thin wire on $[1, 3]$?

$$M = \int_1^3 (2x^2+3)\,dx = \left(\frac{2x^3}{3} + 3x\right)\Big|_1^3 = (18+9) - \left(\frac{2}{3}+3\right)$$

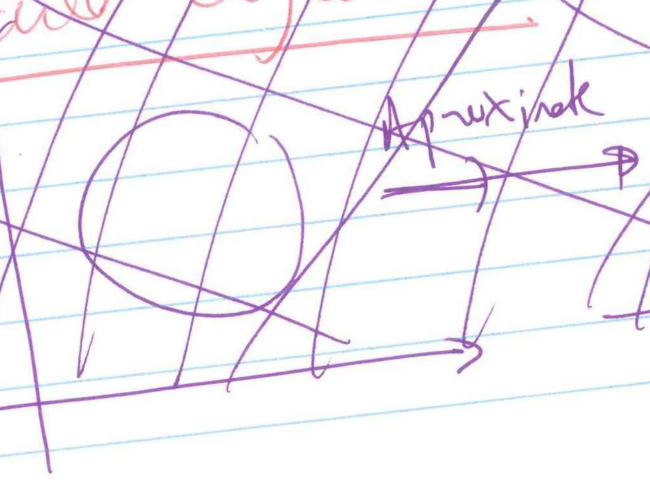
$= 74/3$

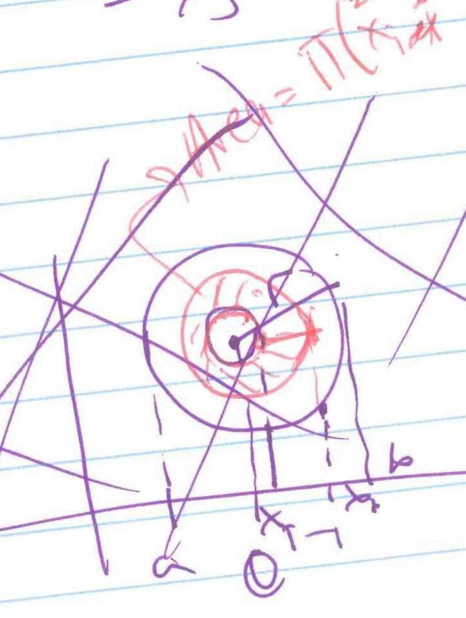
SKIPPED!

For circular objects

Assume ρ is a function of x.

ρ: Area density





Work Done by a Force

Work = Force × distance

But if Force is variable, a function of x, then we need to approximate

- Suppose a variable force $F(x)$ moves an object in positive direction along x axis, from $x=a$ to $x=b$.

To calculate the work done, as before:

- Partition the interval $[a,b]$ into very small parts (of length Δx)
- For each interval $[x_{i-1}, x_i]$, find $W(x_i^*)$

By $W_i \approx F(x_i^*) \cdot \Delta x$.

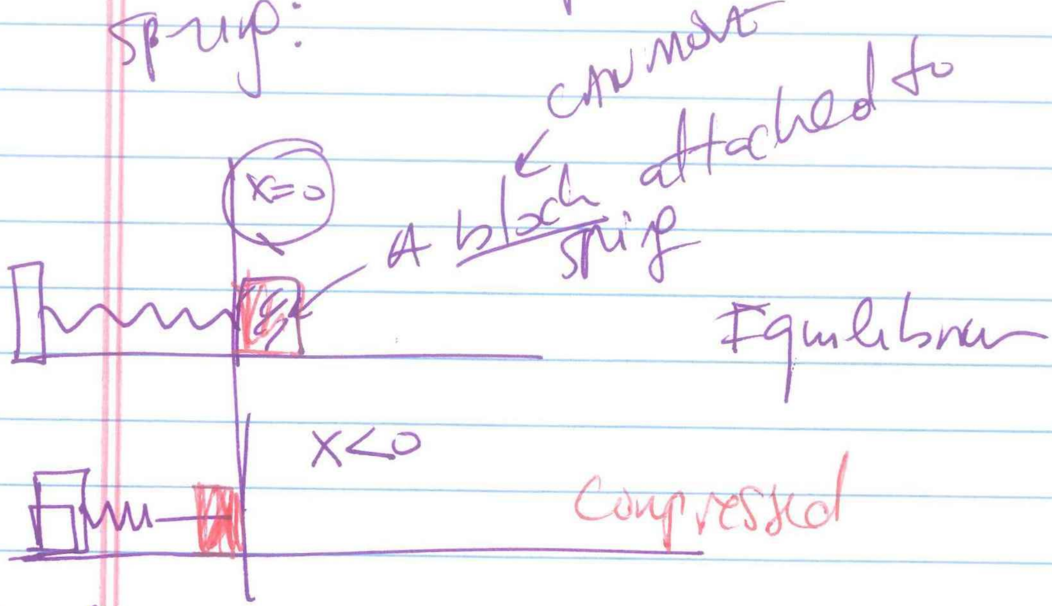
ADD them up

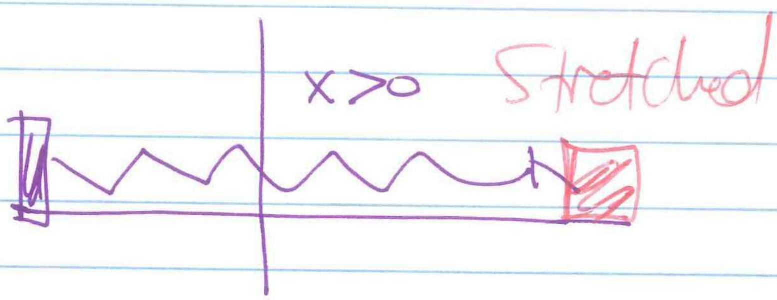
Take limit $n \to \infty$.

$$W = \int_a^b F(x)\,dx$$

← Work done by moving an object in pos direction along x-axis

EX Work done to compress or stretch a spring:





The force required to compress or stretch a spring:

$F(x) = k \cdot x$

↑ spring constant

Work done to stretch or compress a spring:

EX Suppose it takes a force of <u>10 N in the neg. direction</u> to compress a spring 0.2 m from the equilibrium position.

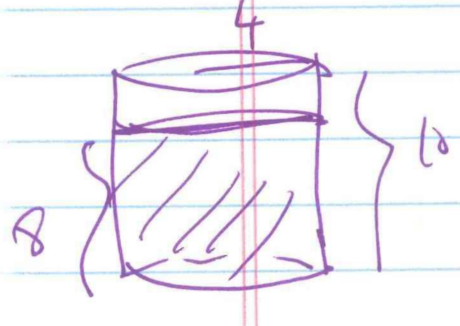
How much work is done to stretch the spring 0.5 m from the equilibrium position?

$$-10N = k\cdot(-0.2) \Rightarrow k = 50.$$

$$W = \int_a^b F(x)\,dx = \int_0^{0.5} 50\cdot x\,dx = 6.25$$

WORK DONE IN PUMPING

Assume we have a cylindrical tank. radius 4m, height = 10 m, filled to a depth of 8m



How much work does it take to pump all the water over the top edge of the tank?