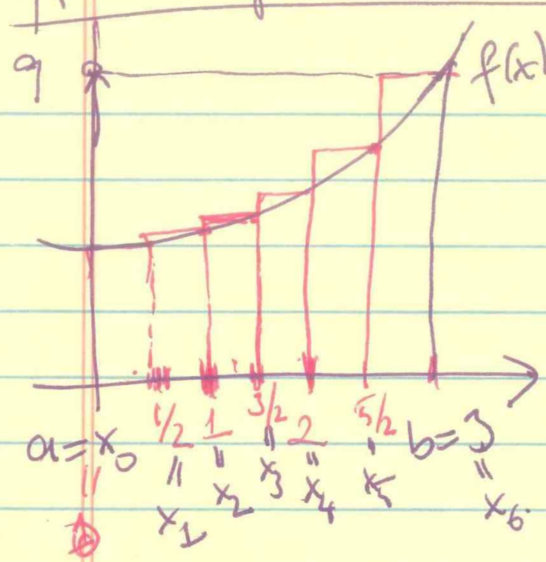
Approximating Areas & Definite Integral:

Use Right endpt



interval $= I = [0,3]$

Divide the region $I=[0,3]$ into 6 intervals

So; $\Delta x = 0.5$

- Calculate the area of each rectangle:

height $= f(x_{i+1}) = (x_{i+1})^2$

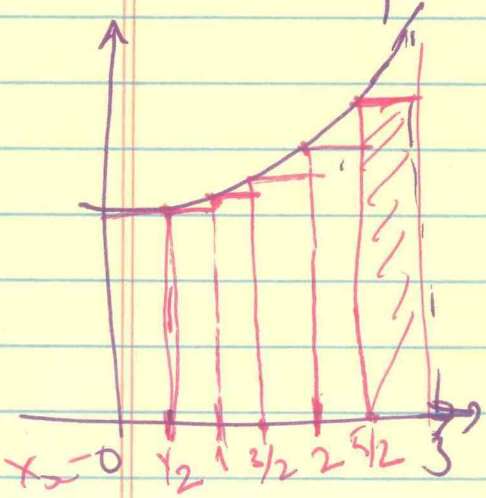
So; Total Area $\approx \sum_{i=0}^{6} f(x_{i+1})\cdot \Delta x_i$

$= \sum_{i=0}^{6} (x_{i+1})^2 \cdot \frac{1}{2}$

So; $(x_1)^2 \cdot \frac{1}{2} + (x_2)^2 \cdot \frac{1}{2} + \cdots + (x_6)^2 \cdot \frac{1}{2}$

$= \frac{1}{4}\cdot\frac{1}{2} + (1)^2\cdot\frac{1}{2} + \left(\frac{3}{2}\right)^2\cdot\frac{1}{2} + (2)^2\cdot\frac{1}{2} + \left(\frac{5}{2}\right)^2\frac{1}{2} + (3)^2\cdot\frac{1}{2}$

≈ 5.6



Use left endpt:

$(x_0)^2\cdot\frac{1}{2} + (x_1)^2\cdot\frac{1}{2} + \cdots + (x_5)^2\cdot\frac{1}{2}$

$= (0)^2\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2\cdot\frac{1}{2} + \cdots + \left(\frac{5}{2}\right)^2\cdot\frac{1}{2}$

≈ 3.4

$\boxed{Q}$ What if we make widths smaller and smaller? (By taking slimmer rectangles. In other words, by dividing I into more rectangles)

Answer: For ex. $R_{32} = 8.06$ & $L_{32} = 7.92$

So: Right & Left estimates get closer to each other!!!

Lets do it for n.

Area under the curve:

$f(x)$: Continuous, non-neg. function on $[a,b]$

$\sum_{i=1}^{n} f(x_i)\Delta x$: Riemann sum for $f(x)$

Then, area under curve is given by:

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x}$$

DEFINITE INTEGRAL OF f from a to b

$$= \int_a^b f(x)\,dx$$

FUNDAMENTAL THEOREM OF CALCULUS

Part I

If $f(x)$ is continuous on an interval $[a,b]$, ~~then~~ and $F(x) = \int_a^x f(t)\,dt$,

then $F'(x) = \frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$ over $[a,b]$

Part II

If f is continuous over the interval $[a,b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Ex: Evaluate $I = \int_{-2}^{2} (t^2-4)\,dt$

An Antiderivative of t^2-4 is: $\frac{t^3}{3} - 4t + 1 = F(t)$

So, $I = \left[\frac{(+2)^3}{3} - 4.(+2) + 1\right] - \left[\frac{(-2)^3}{3} - 4(-2) + 1\right]$

$= -32/3.$