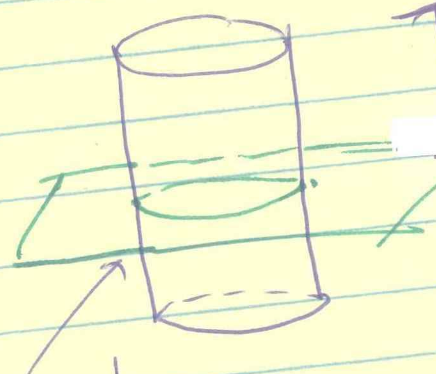
DETERMINING VOLUMES BY SLICING:

In general,

$$V = \underbrace{\text{length} \times \text{width}}_{\text{Area of (a (slice)) bottom}} \times \text{height}$$

Idea:



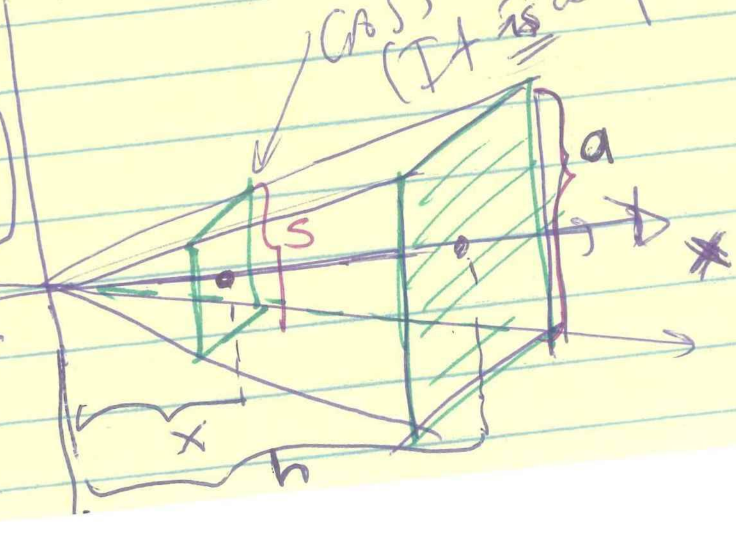
To find volume:
slice into very thin pieces;
Calculate the areas
integrate from bottom to top
(= dx or dy)
Thickness

cross-section
(it is a disk)

So, $V \approx \sum (\text{Areas of slices}) \Delta x$ Thickness

$\boxed{\text{Ex}}$ Pyramide:

cross-section
(It is a square)



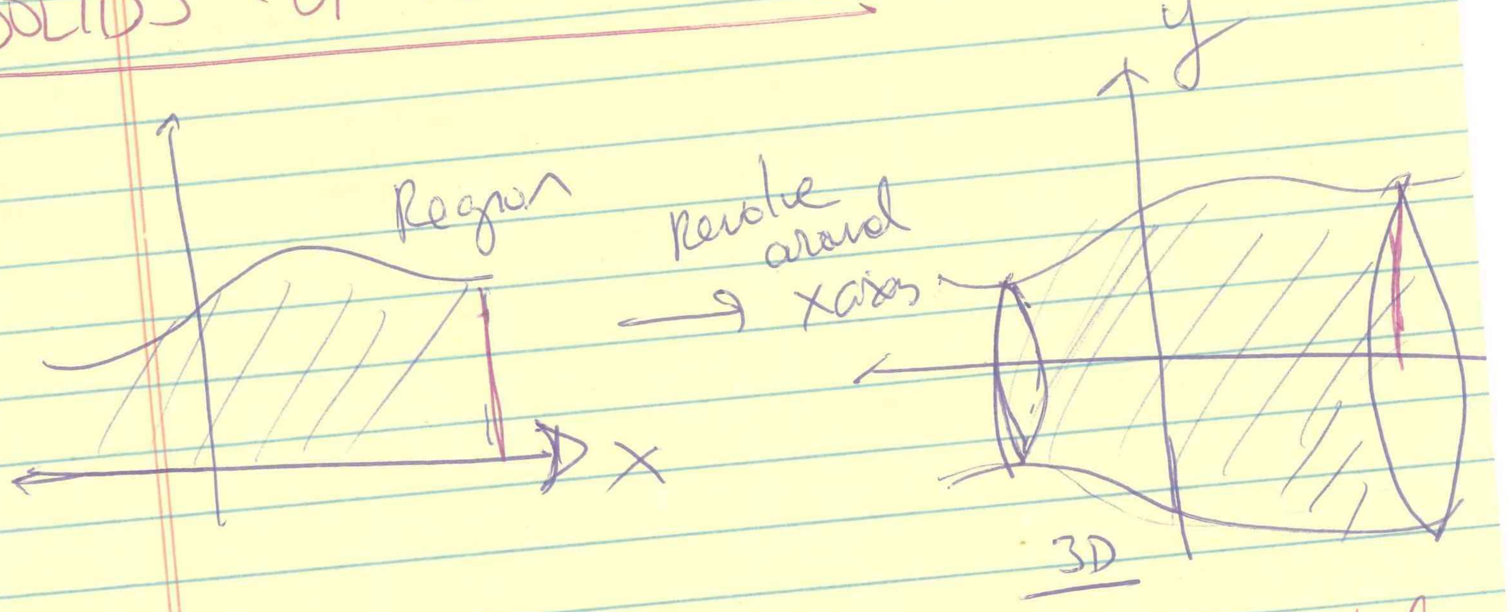
$$\boxed{\frac{x}{h} = \frac{s}{a}} \to \boxed{s = \frac{ax}{h}}$$

Area of slice $= s^2 = \left(\frac{ax}{h}\right)^2$

$$\Rightarrow V = \int_0^h A(x)\,dx = \int_0^h \left(\frac{ax}{h}\right)^2 dx = \frac{a^2}{h^2}\int_0^h x^2\,dx$$

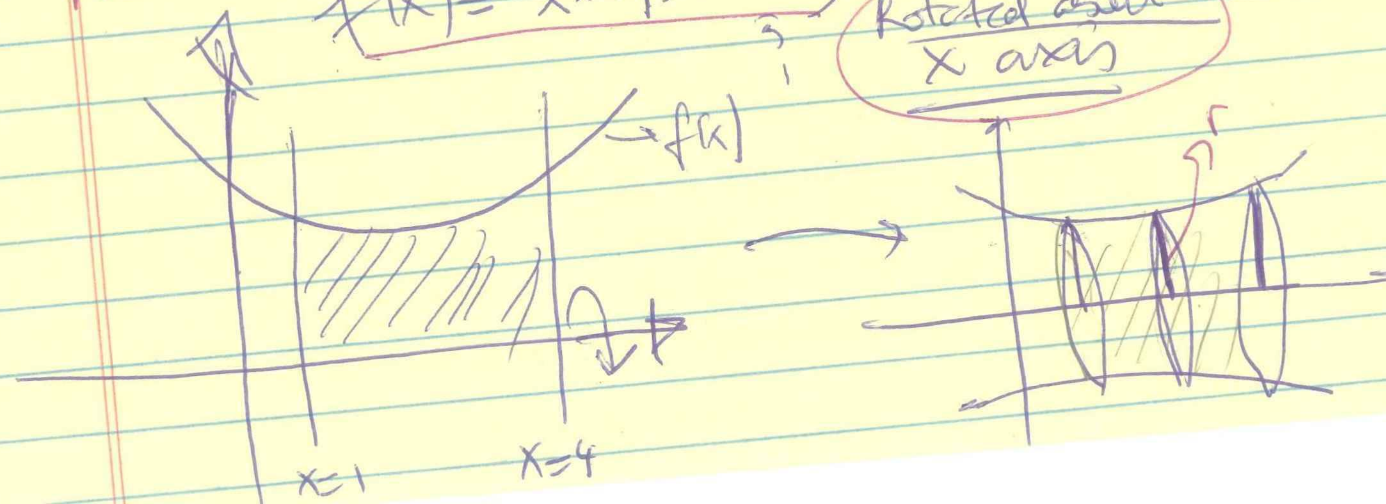
$$= \frac{1}{3}a^2 h$$

SOLIDS OF REVOLUTION:



$\boxed{EX}$ Using slicing method, find volume of solid of revolution given by: $f(x) = x^2 - 4x + 5$, $x = 1$, $x = 4$.

Rotated about x axis



Area of a slice = πr^2

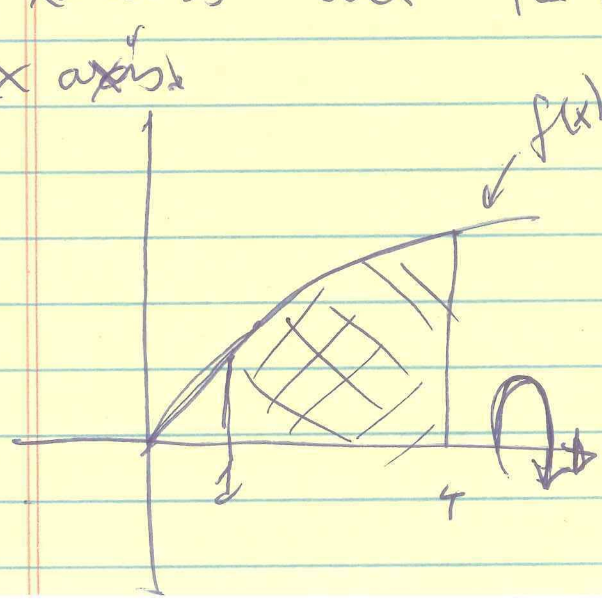
r is given by $f(x)$. So,

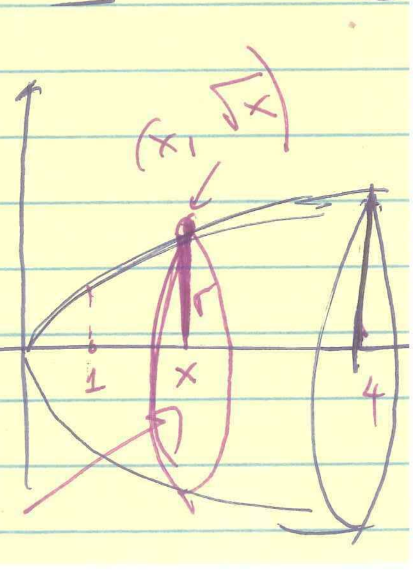
$$V = \pi \int_1^4 (x^2 - 4x + 5)^2 \, dx$$

slicing along x axis,

This is also called "DISK METHOD" because our slices are disks.

$\boxed{EX}$ Use the disk method to find the volume of the solid of revolution generated by rotating the region between the graph of $f(x) = \sqrt{x}$ and the x-axis over the interval $[1,4]$ around the x axis.





Area $= \pi r^2$

$$\Rightarrow V = \int_1^4 \pi r^2\,dx = \pi \int_1^4 (\sqrt{x})^2\,dx = \frac{15}{2}\pi.$$

What if we revolve around y axis?

$\boxed{\text{Ex}}$ Let R be the region bounded by the graph of $g(y)=\sqrt{4-y}$ and the y-axis over the y-axis interval $[0,4]$.

Use the disk method to find the volume of the solid of rev. generated by rotating R around the y axis.

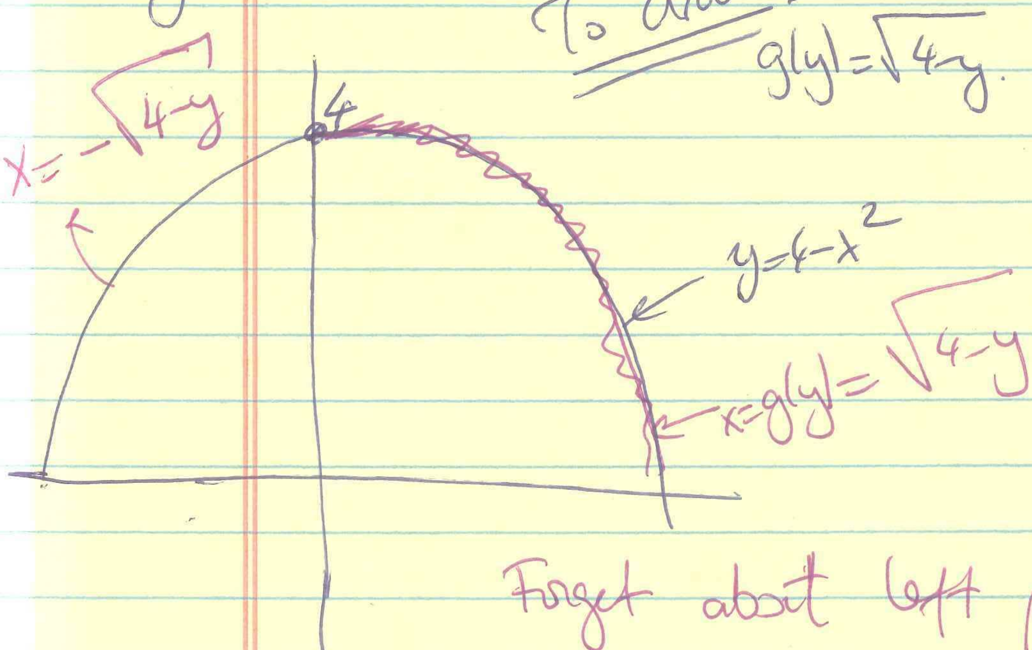
To draw: $g(y)=\sqrt{4-y}$.

$x=\sqrt{4-y}$

$\Updownarrow$

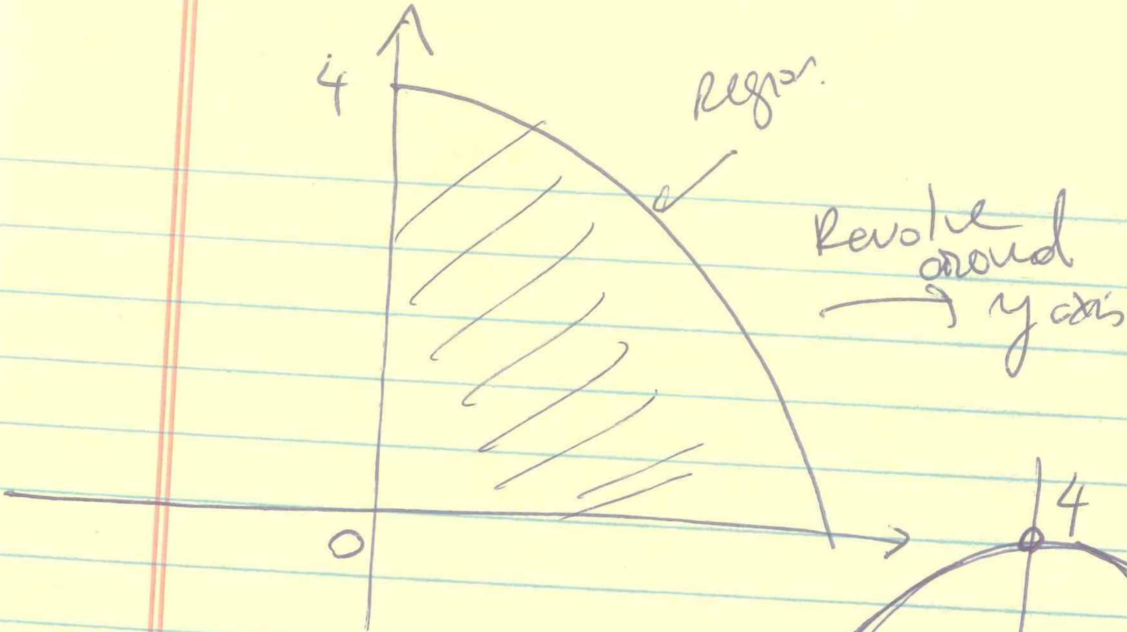
$x^2=4-y.$

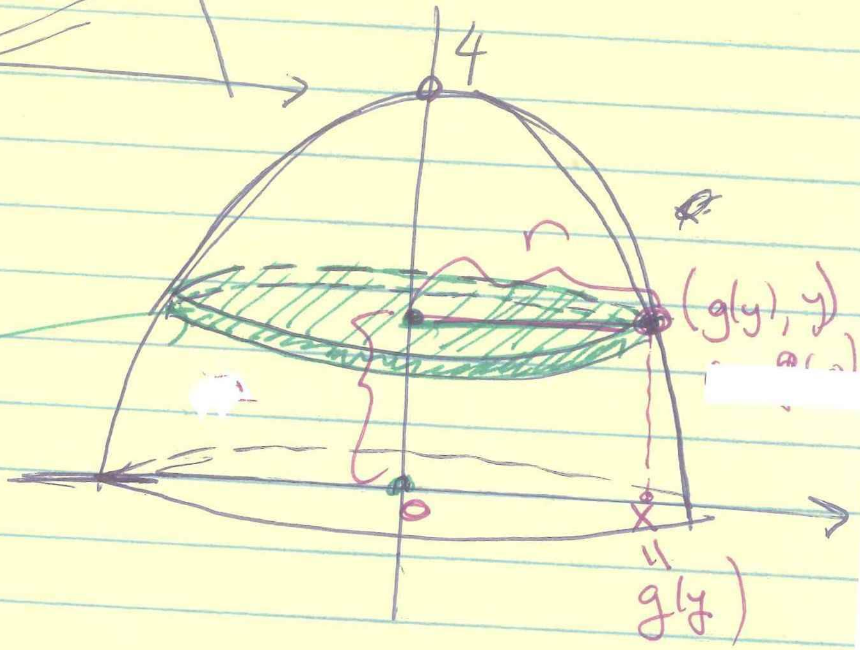
$\boxed{y=4-x^2}$



Forget about left part of the graph.

$\longrightarrow$





A cross section is a disk:

$$\text{Area} = \pi r^2$$

$$V = \int (\text{Area}) * \text{thickness}$$

$$= \int_0^4 \underbrace{[\text{Area}]}_{? = \pi r^2} \overset{\text{times}}{*} dy$$

But, what is r?
(In terms of y)

$$V = \pi \int_0^4 (\sqrt{4-y})^2 \, dy = \pi \int_0^4 (4-y)\, dy = 8\pi$$