Application of row-reduction of operator matrices to the computation of π-flat outputs in control theory

Felix Antritter Johannes Middeke

March 2011

In applied control theory, *differential flatness* is an important tool for the analysis of linear differential systems. See, for example, [8, 6] for definitions and [9, 7] for applications of this concept. In brief, this system property is characterised by the existence of a *flat output*, that allows a parameterisation of the states and inputs of a system using linear combinations of derivatives of the flat output.

The concept of π-flatness is an extension to linear control systems with delays. A π-flat output allows to parameterise the states and inputs of the considered system using linear combinations of the derivatives of this output and, additionally, predictions, which are characterised by a prediction operator π. Of the several proposed extensions—see, for example, [11, 13, 10] and see also [14, 3] for a different approach—we have have chosen to elaborate on the method that is discussed in [10] which is based on the concept of so-called *hyper-regularity* of matrices.

We model linear time-varying control systems with delays by *Ore polynomials* in a differential operator $\frac{d}{dt}$ and a delay (shift) operator δ with coefficients originating from a field K of time-varying functions—cf [12, 5, 4] for a definition of (iterated) Ore polynomial rings. Linear time-varying systems with delays are then of the form

$$Ax = Bu$$

with $A \in K[\delta, \frac{d}{dt}]^{n\times n}$ and $B \in K[\delta, \frac{d}{dt}]^{n\times m}$ being matrices of differential and delay operators. The system is π-flat for $\pi \in K[\delta]$ if there are matrices $P \in K[\delta, \frac{d}{dt}]^{m\times n}$, $Q \in K[\delta, \frac{d}{dt}]^{n\times m}$ and $R \in K[\delta, \frac{d}{dt}]^{m\times m}$ such that

$$y = \pi^{-1}Px, \qquad x = \pi^{-1}Qy \qquad \text{and} \qquad u = \pi^{-1}Ry$$

where y is the flat output.

As in [10] we localise the ring $K[\delta, \frac{d}{dt}]$ at the non-zero polynomials in δ. The resulting ring $K(\delta)[\frac{d}{dt}]$ can then be regarded as Ore polynomial ring in the single variable $\frac{d}{dt}$. In [10], it was proposed to check the hyper-regularity of matrices of operators by computation of the Smith-Jacobson normal form which may be computed using an algorithm sketched in that paper. In our contribution, we give a different characterisation of hyper-regular matrices that allows us to use the efficient method of *row-* and *column-reduction* (cf, for example, [2]) in order to check the considered matrices for hyper-regularity. This yields an algorithm which has been implemented in a Maple™ toolbox by one of the authors that can be downloaded at [1].

References

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