4.2 Mean Value Theorem:
Theorem: Suppose

1. $f(x)$ is continuous, $a \leq x \leq b$ and
2. $f(x)$ is differentiable, $a < x < b$.

Then there is number c, $a < c < b$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Picture:

Application: Suppose the $f(x)$ is a continuous function such that $f'(x) = 0$, $c \leq x \leq d$. Then $f(x)$ is constant.

Explanation: For suppose $a < b$: If $f(b) \neq f(a)$ then

$$\frac{f(b) - f(a)}{b - a} = f'(c) \neq 0$$

Example: Show that $x^3 + 3x + 2 = 0$ has exactly one solution.

Solution: If $f(x) = x^3 + 3x + 2$ then $f(-1) = -2 < 0$ but $f(0) = 2$ so that $f(x) = 0$ has at least one solution between $x = -1$ and $x = 0$ by the intermediate value theorem. Since $f'(x) = 3x^2 + 3 > 0$, whenever $b > a$ $f(b) > f(a)$ so the graph of f rises and rises and can never fall. So it crosses $y = 0$ only once.