11.3 Polar Co-ordinates:

This is an alternate way of specifying points on the plane. It is useful when there is a distinguished point (e.g the sun). Instead of the xy coordinate axes there is a point and ray and position is given by specifying the distance r to the point and the angle θ formed with the ray. Picture

Example: The point with polar coordinates $(r, \theta) = (3, \pi/3)$ is here. One can recover the Cartesian coordinates with the conversion

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

or in this case $x = 3/2$ and $y = 3\sqrt{3}/2$. Conversely the point $(3/2, 3\sqrt{3}/2)$ in Cartesian coordinates has polar coordinates $(r, \theta) = (3, \pi/3) = (3, \pi/3 + 2n\pi)$ where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ or in fact $(r, \theta) = (3, \pi/3) = (-3, 4\pi/3 + 2n\pi)$. However, it is true that

$$r^2 = x^2 + y^2$$

(An equation for θ is trickier.)

11.4 Graphing in Polar Co-ordinates Example: Graph the curve given in polar coordinates by

$$r(\theta) = 3(1 + \cos\theta)$$

Solution 1: It is possible to plot points

θ	$r = 3(1 + \cos\theta)$
0	6
$\pi/6$	$3(1 + \sqrt{3}/2) \sim 5.59$
$\pi/4$	$3(1 + \sqrt{2}/2) \sim 5.12$
$\pi/3$	$9/2 = 4.5$
$\pi/2$	3
$2\pi/3$	$3/2 = 1.5$
π	0
$\vdots$	$\vdots$

but it is faster to work from a plot of $y = 3(1 + \cos x)$.

Solution 2: Graph of $y = 3(1 + \cos x)$ in Cartesian coordinates!

Then use the plot as if it were a table of values to get the polar plot.

Example: Plot the curve $r = a \sin\theta$ where $a > 0$ is a constant.

Convert to Cartesian coordinates:Multiply by r, $r^2 = ar\sin\theta$ so that $x^2 + y^2 = ay$ and then complete the square: $x^2 + (y - a/2)^2 = a^2/4$ Its a circle of radius $a/2$ centered at $(0, a/2)$ (in Cartesian coordinates).

Example: Similarly $r = a\cos\theta$ is a circle of radius $a/2$ and center $(a/2, 0)$.

Example: Sketch the curve in polar coordinates $r = 2\sin 3\theta$.

Solution. Sketch in Cartesian coordinates $y = 2\sin 3x$. It is enough to graph for $0 \le x \le 2\pi$ because $2\sin 3x$ has period 2π as does θ of polar coordinates.

Use this instead of a table of values for $r = 2\sin 3\theta$. The picture is a 3 leaved rose.