11.1 Parametric Curves: Until now curves have always been specified as the graph of a function $y = f(x)$. This is because the graph is used to study f. Now we are interested in studying general curves. We do not want to be restricted by the vertical line test. How does one specify a general curve? As points $(x(t), y(t))$ traced out as time t passes, $a \leq t \leq b$.

Example: Graph the curve

$$x(t) = 3\cos t, \quad y = 3\sin t, \quad 0 \leq t \leq 4\pi$$

Solution. In desperation we plot points

t	$x = 3\cos t$	$y = 3\sin t$
0	3	0
$\pi/6$	$3\sqrt{3}/2$	$3/2$
$\pi/4$	$3\sqrt{2}/2$	$3\sqrt{2}/2$
$\pi/3$	$3/2$	$3\sqrt{3}/2$
$\pi/2$	0	3
$2\pi/3$	-3/2	$3\sqrt{3}/2$
$\vdots$	$\vdots$	$\vdots$

It looks like a ...? A circle. Eliminate t: $x(t)^2 + y(t)^2 = 9(\cos t)^2 + 9(\sin t)^2 = 9$. It's a circle of radius 3 centered at the origin. It is traced out twice as t increases from 0 to 4π because $\sin t$ and $\cos t$ are periodic with period 2π.

Example: Identify the curve $x = 3\cos 2\pi t$, $y = 3\sin 2\pi t$ $0 \leq t \leq 2$. It is the same curve as above but it is traced out 2π times as fast.

Example: Identify the curve $x = 3\sin t$, $y = 3\cos t$ $0 \leq t \leq 2$. It is the same curve as above but it is traced out clockwise starting form the north pole.

Example: Identify the curve $x = 5\cos t$, $y = 3\sin t$ $0 \leq t \leq 2\pi$.

Solution: Eliminate t:

$$\begin{aligned} \left(\frac{x}{5}\right)^2 + \left(\frac{y}{3}\right)^2 &= 1 \\ \frac{x^2}{25} + \frac{y^2}{9} &= 1 \end{aligned}$$

It's an ellipse with major axis 5 on the x-axis and minor axis 5 on the x-axis. Sketch:

Example: Identify and sketch the curve $x = 1 + 5\cos t$, $y = 2 + 5\sin t$

Solution: This is just the previous curve shifted one unit right and two units up.

Example: Parameterize the curves:

1. $y = x\cos x + x^2, \quad 0 < x < 3\pi$

2. The straight line segment from (1,2) to (5, -3).

Solution: These are the types of curves we could already treat as the graph of a function. We can, of course also treat them as parameterized curves.

1. $x(t) = t$, and $y(t) = x \cos x + x^2 = t \cos t + t^2, \qquad 0 < t < 3\pi$
2. $x(t) = 1 + 4t$, $y(t) = 2 - 5t$ $0 \leq t \leq 1$