Theorem 1. *(Theorem 6.15) Suppose A is a set with an equivalence relation defined on it. For each $a \in A$ let $\overline{a} = \{b \in A : b \sim a\}$ which is the equivalence class that contains a. Then $\overline{a_1} \cap \overline{a_2} \neq \emptyset$ if and only if $a_1 \sim a_2$ if and only if $\overline{a_1} = \overline{a_2}$. Then A is the union of equivalence classes*

$$A = \cup_a \overline{a}$$

Proof. Suppose $\overline{a_1} \cap \overline{a_2} \neq \emptyset$ so that there is $b \in \overline{a_1} \cap \overline{a_2}$ so that $b \sim a_1$ and $b \sim a_2$. This says $a_1 \sim a_2$ by symmetry and transitivity. Conversely $a_1 \sim a_2$ implies $a_1 \in \overline{a_2}$ so that $a_1 \in \overline{a_1} \cap \overline{a_2}$. Finally we observe that if $\overline{a_1} \cap \overline{a_2} \neq \emptyset$ then $a_1 \sim a_2$ and if $b \in \overline{a_1}$ the $b \sim a_1$ and so $b \sim a_2$ by transitivity and so $b \in \overline{a_2}$ and this proves that $\overline{a_1} \subseteq \overline{a_2}$. The converse $\overline{a_2} \subseteq \overline{a_1}$ follows by a symmetric argument. The remaining conclusion $A = \cup_a \overline{a}$ is evident. $\square$