Assignment 9, Math 4820\5820
Due December 9

1. Suppose that $a_n \geq 0$ for all $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} a_n$ converges. Prove or disprove

   (a) $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n}$ converges.

   (b) $\displaystyle\sum_{n=1}^{\infty} a_n^2$ converges.

   (c) $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}}$ converges.

2. Give an example of a sequence a_n, $n \in \mathbb{N}$ with the following property: For every $p \in \mathbb{N}$ and every $\epsilon > 0$ there exists $N \in \mathbb{N}$ so that $|a_m - a_n| < \epsilon$ for all $n \geq N$ and m, $n \leq m \leq n + p$ but a_n is not a Cauchy sequence. (Suggestion: Try $a_n = \sqrt{n}$.)

3. Compute $\limsup_{n\to\infty} n \sin n$. Justify your answer.

4. (I) Find the radius of convergence of the series

   (a) $\sum_{n=0}^{\infty} \dfrac{n!}{5^n}(x-4)^n$

   (b) $\sum_{n=0}^{\infty} \dfrac{2^n}{n^n}(x+3)^n$

   (c) $\sum_{n=0}^{\infty} \dfrac{3^n}{n}(x-1)^n$

   and (II) Check each series for convergence at the end points of the interval of convergence.