Assignment 7, Math 4820\5820

Due November 16

1. Exhibit an open cover of $[1,2)$ which has no finite subcover.

2. Show that if K_1 and K_2 are compact sets in a metric space X then $K_1 \cup K_2$ is also compact.

3. (a) Suppose that $f : [0,1] \to \mathbb{R}$ is a function with the property that for every a, $0 \le a \le 1$ there is an open set V_a containing a and $B_a > 0$ so that $|f(x)| < B_a$ for all $x \in V_a$. Show that there exists $B > 0$ so that $|f(x)| < B$ for all x, $0 \le x \le 1$.

   (b) Suppose that $f : (0,1] \to \mathbb{R}$ is a function with the property that for every a, $0 < a \le 1$ there is an open set V_a containing a and $B_a > 0$ so that $|f(x)| < B_a$ for all $x \in V_a$. Show that f need not be bounded: That is (possibly) there is no $B > 0$ so that $|f(x)| < B$ for all x, $0 < x \le 1$.

4. Suppose that a_n is a sequence in $\mathbb{R}^k$ and $\lim_{n\to\infty} a_n = A$ exists. Show that $|a_n|$ converges and $\lim_{n\to\infty} |a_n| = |A|$. Is it necessarily true that if a_n is a sequence such that $|a_n|$ is convergent then a_n also converges?

5. Suppose that s_n is a convergent sequence in X. Show that every subsequence of s_n is also convergent and that all subsequences have the same limit.