

Assignment 4, Math 4820\5820
Due Monday, October 17

1. Define a relation R on the real numbers \mathbb{R} as xRy if and only if $x - y \in \mathbb{Q}$

- (a) Show that R is an equivalence relation.
- (b) Describe the equivalence class that contains $\sqrt{2}$
- (c) What is the cardinality of that equivalence class?
- (d) How many equivalence classes are there?

(This equivalence relation is often used to construct a non-measurable set.)

2. Let $H = \{1/n : n \in \mathbb{N}\}$

- (a) Construct a sequence of open sets G_n in \mathbb{R} so that $H \subseteq \bigcup_{n=1}^{\infty} G_n$ but there is no m so that $H \subseteq \bigcup_{n=1}^m G_n$
- (b) Show that if $G_n, n \in \mathbb{N}$ is a sequence of open sets in \mathbb{R} so that $H \cup \{0\} \subseteq \bigcup_n G_n$ then there is $m \in \mathbb{N}$ so that $H \cup \{0\} \subseteq \bigcup_{n=1}^m G_n$. (Try to avoid using the Heine Borel Theorem.)

3. Find E' , the set of all limit points of the set $E = \{x \in \mathbb{Q} : 0 < x < 1\}$. (E is regarded as a subset of \mathbb{R} .) What is the closure \overline{E} of E ?