

Assignment 4, Math 4820\5820

Due Friday, October 7

1. Show that every infinite set contains a copy of  $\mathbb{N}$ . More precisely show that, if  $A$  any infinite set then there is an injection of  $\mathbb{N}$  into  $A$ . (This shows that the cardinality of  $\mathbb{N}$  (written  $\aleph_0$ ) is the smallest infinite cardinal.)
2. Show that if  $A$  and  $B$  are two disjoint finite sets then the cardinality of  $A \cup B$  is the sum of the cardinalities of  $A$  and  $B$ .
3. Show that a set  $A$  is infinite if and only if  $A \sim B$  where  $B$  is some proper set of  $A$ .
4. Read Theorem 8.18 of Lay's Section 8 on Cardinality (posted on our website). Read also the subsequent discussion. Briefly explain in your own words why there is no maximal cardinal and yet we do not know what the "next" cardinal after  $\aleph_0$  is. You should mention the continuum hypothesis.
5. Problem 17, page 23 from our text: Prove that

$$|\mathbf{x} + \mathbf{y}|^2 + |\mathbf{x} - \mathbf{y}|^2 = 2|\mathbf{x}|^2 + 2|\mathbf{y}|^2$$

if  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^k$ . Interpret this geometrically as a statement about parallelograms.