

Assignment 3, Math 4820\5820  
Due Friday, September 23

1. Define the relation  $R$  on the set  $\mathbb{Z}$  of integers by  $xRy$  iff  $x - y = 5k$  for some integer  $k$ . Thus  $xRy$  iff  $x - y$  is divisible by 5.
  - (a) Show that  $R$  is an equivalence relation.
  - (b) Describe its five distinct equivalence classes.
2. Define the relation  $R$  on the set  $\mathbb{Z}$  of integers by  $xRy$  iff  $x + y = 5k$  for some integer  $k$ . Is  $R$  an equivalence relation?
3. Let  $A = \{1, 2\}$  and  $B = \{a, b, c\}$ .
  - (a) Determine the number of injections of  $A$  into  $B$ .
  - (b) Determine the number of surjections of  $B$  onto  $A$ .
4. Let  $f : A \rightarrow B$  and  $g : B \rightarrow A$  be two function and suppose that  $g \circ f = i_A$  and  $f \circ g = i_B$ . (Here we use the notation  $i_A : A \rightarrow A$  is the “identity” function such that  $i_A(a) = a$  for every  $a \in A$ .) Prove that  $f$  is bijective. Check that  $g$  is also bijective. We call  $g$  the inverse of  $f$  (when it exists) and write  $g = f^{-1}$ . (But do not confuse this notation with the notation  $f^{-1}$  defined on page 24 for all functions.)
5. Recall that the power set  $\mathcal{P}(A)$  of a set  $A$  is the set of all subsets of  $A$ . List the elements of  $\mathcal{P}(\{1, 2, 3\})$ . Show that if  $A$  has  $n$ -elements then  $\mathcal{P}(A)$  has  $2^n$  elements. (You may use induction on  $n$ .)
6. If  $A$  has  $m$  elements and  $B$  has  $n$  elements then how many elements does the Cartesian product  $A \times B = \{(a, b) : a \in A, b \in B\}$  have? (Here  $(a, b)$  is an “ordered pair.”)