History of Mathematics

Math 3510

Text: The History of Mathematics An Introduction, 7th ed., by David M. Burton, McGraw Hill

The history of mathematics is an account of the important ideas and results of mathematics and the people and societies that discovered them.

Some topics are studied by generation after generation of mathematicians. For example, the ancient Babylonians discovered an empirical method for solving quadratic equations (sometime before 1800 BCE) and subsequently higher order polynomial equations were studied (like $ax^3+bx^2+cx+d=0$). Generations worked on teh cubic equation and then the quartic and then the quintic. This study culminated in the brilliant works of Niels Henrik Abel (1802-1829), and Evariste Galois (1811-1832), two brilliant, fascinating and tragic figures among mathematicians who were able to solve the ... but showed that there can be no solution of the ... using the historic methods (like extraction of roots).

The history of mathematics is a mathematics and history both. To understand the historical development of an idea (like solving polynomial equations) one must know what was known by whom and who contributed to our knowledge. One outcome of studying the history of math is a better understanding of what math is. For example one of the most famous results of the last 30 years is the resolution of Fermat's last theorem (Pierre de Fermat 160?-1665) by Andrew Wiles (1993-1995). (The theorem is described below.) Why was that such a milestone?

Many accuse western historians of bias: highlighting the work of Europe, Greece, Babylon and Egypt over China, India and southest Asia. It is wise to admit ignorance. The dry climate of ancient Egypt and the record keeping of the ancient Egyptians and Babylonians allows us to look more deeply into history of mathematics there than in India or China, for example. Perhaps future archaeologists will make discoveries that will help us see into other civilizations. In looking so far back into the past it will be necessary to not only discuss what little we know about the ancient civilizatons but also how we know it. For example, we are dependent on people like Jean François Champollion (1790-1832) for deciphering ancient dead languages and to Napoleon Bonaparte (1769-1821) for encouraging French and European scholars to study ancient Egypt. As a second example, Pythagoras is attributed with the Pythagorean theorem but we really know very little about Pythagoras or his school and certainly acquaintance with instances of the Pythagorean theorem goes back a thousand years before Pythagoras. Was he the first to state his famous theorem $a^2+b=c^2$?

Some recent developments however are comprehensible at least superficially and they remind us that mathematics is a continuing endeavor and that we and our students may participate in its development.

Fermat's Last Theorem Pierre de Fermat 160?-1665 wrote in the margin of his copy of Diophantus's *Arithmetica* that he had found a marvellous proof of the following (in modern notation)

There are no positive integers $n \geq 3$, x, y, z so that

$$x^n + y^n = z^n$$

Fermat then said there was no space in the margin to write the proof. It is believed that Fermat could prove the case $n = 4$ by his method of "infinite descent" but that he mistakenly believed his method worked for all $n \geq 3$ but it does not. Credit for the proof of Fermat's Last Theorem is given to Andrew Wiles who in 1993 published a proof which was found to have an error which was corrected with the help of Richard Taylor in 1995 in the *Annals of Mathematics*. Indeed Wiles proved the "modularity theorem" which is difficult to describe but is said to have consequences beyond Fermat's Last Theorem.

Kepler's Conjecture: Johannes Kepler (1571-1630) in 1611 stated without proof that the densest packing of 3 dimensional balls of common radius is face centered cubic packing familiar to us at fruit stands and in stacks of cannonballs. Tom Hales verified the conjecture in a 250 page article which involved considerable computer checking. The article was submitted in 1998 (roughly) and has been reviewed and was scheduled in 2003 for publication in *Annals of Mathematics.* Reviewers agreed that they thought the proof was correct but felt that the proof was so complicated they still had reservations. Hales proof also extends to balls in any finite number of dimensions.

Hopf Conjecture: Heinz Hopf conjectured (1951?) that any surface with constant mean curvature must be a sphere. His conjecture was verified under the additional assumption that the surface have no self intersections (embedded in $\mathbb{R}^3$). However if self intersections are allowed (an immersed surface in $\mathbb{R}^3$) then a counterexample was constructed by Henry Wente in 1986 in Pacific Journal of Mathematics. In fact his method constructs an infinite family of such surfaces.

Poincare Conjecture: Conjecture (H. Poincare, 1904): Every compact bound- e aryless simply connected 3-dimensional manifold is homeomor- phic to the 3-dimensional unit sphere S^3 . This conjecture is posed by Henri Poincaré in 1904 (original e statement was slightly different) and solved by Grigory Perelman in 2002. First Clay Mathematics Institute Millennium Prize Announced March 18, 2010 the Prize for Resolution of the Poincar Conjecture Awarded to Dr. Grigoriy Perelman

Henri Poincar

The Clay Mathematics Institute (CMI) announces today that Dr. Grigoriy Perelman of St. Petersburg, Russia, is the recipient of the Millennium Prize for resolution of the Poincar conjecture.

Four Color Map Problem Every map, no matter how intricate of "countries" on a sphere or a plane requires at most 4 colors so that adjacent countries are not the same color. Kenneth Appel and Wolfgang Haken in 1976 used computers to check many cases.

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1.2 **Egyptian Number System**

Items of Special Interest from the Text:

1. Herodotus (485-430 B.C.) and the book *History of Herodotus.*

2. King Narmer of Egypt (ca 3000 B.C?) and the Narmer Palette of slate and a stone macehead that records the spoils of war from the Libyans at Hierakonpolis (Greek Nekhen) south of Luxor (Thebes) on the Nile.

Egypt's dry climate and the people's habit of entombing their dead have allowed us to look back further into Egypt's past than into other cultures'. Upper and Lower Egypt were united in 3100 B.C. by a ruler called Menes. For comparison, the first dynasty is dated as beginning in 2950 B.C. (there were 31 dynasties) and Akhenaton ruled about 1350 B.C. and was (likely) the father of Tutankhamun (1325 B.C?). About 3000 B.C. Egyptian ruler Narmer defeated and exacted tribute from the Libyans of the western Nile delta and the record left in stone (Burton, page 12) enumerates the spoils leaves a record of Egyptian use of large numbers.

$$1 \quad 10 \quad 100 \quad 1000 \quad 10,000 \quad 100,000 \quad 1,000,000 \quad 10,000,000$$

Each new power of ten requires a new symbol. Incidentally, why do the Egyptians and us and many others use base 10 for their numbers? Position of the symbols has no significance.

Example: Add 261,822 to 52,391.

These symbols are straightforward to work with in simple calculations but they are very slow to write down. A more cursive notation "hieratic" was adopted in later generations: see page 13 of Burton.

Ionic Greek Number System: This system came much later in 400-500 B.C. and is on page 14 of the text.

1	α	10	ι	100	ρ
2	β	20	κ	200	σ
3	γ	30	λ	300	τ
4	δ	40	μ	400	υ
5	ϵ	50	ν	500	ϕ
6		60	ξ	600	χ
7	ζ	70	o	700	ψ
8	η	80	π	800	ω
9	θ	90		900	

The symbol for 6 is "digamma" and for 90 is "koppa" and for 900 is the "sampi" borrowed from the Phoenician alphabet for a total of 37 symbols. For numbers larger than 999 ($= \quad \theta$), an accent mark is used to denote thousands $,\zeta\psi o\zeta = 7,777$. To multiply by 10,000, the symbol M is used so that $\upsilon\mu\delta M = 4,440,000$.

Example: Compute 37×68.

$\xi\eta$			68		
$\lambda\zeta$			$\times 37$		
$,\alpha\omega$	$\upsilon\kappa$		1800	420	
$\sigma\mu$	ν	$= ,\beta\phi\iota\nu$	240	56	$= 2516$

1.3 **Babylonian Number System:** Babylonians of Mespotamia (between the Tigris and Euphrates) wrote on clay tablets often the size of a hand usually with a stylus (made of papyrus) that made nails and wedge shapes in the wet clay. When baked the tablets lasted for ages; several are from 2000 B.C. Read the account "Number Recording of the Babylonians" of the text, the discovery of cuneiform writing and the table of squares of the of natural numbers up to 59 and cubes up to 32. (Cuneus is wedge in Latin.)

Numerals were written in the sexagesimal system (base 60 to a mathematician). It is a positional system to a certain extent but there was no zero and no decimal point.

Example: The Babylonian number

corresponds to our $1 \cdot 60^3 + 23 \cdot 60^2 + 51 \cdot 60 + 47 = 301,907$. However because there is no zero at least initially, the meaning of a number could be ambiguous. For example, the Babylonian number

could mean 134 or 7214 or 8040 depending on whether one interprets a zero between the two groupings or after them. In later times (300 B.C.) the symbol was used as a placeholder. One could also interpret the same Babylonian number above to have a decimal, in which case it might be 2 + 7/30.

Our text makes some remarks about mathematics and science in China (pp 25-26) such as the invention of paper in A.D. 105 (as opposed to in Spain in 1150) and there are number systems (Chinese and Mayan) discussed in the problems (pp 26-29).

Claudius Ptolemy (c. AD 90 c. AD 168), was a Roman citizen of Egypt who wrote in Greek. He was a mathematician, astronomer, geographer, astrologer, and poet (of a single epigram in the Greek Anthology). He lived in Egypt under Roman rule, and is believed to have been born in the town of Ptolemais Hermiou in the Thebaid. He died in Alexandria around AD 168. His most famous work is the *Amalgest* where he announces that he will do his calculations in the sexagesimal system to avoid the embarrassment of [Egyptian] fractions.