Two Pages! **Quiz 9B**, Math 1860-022
4/17/14 Solutions Name ___________________

1. (4) Find the length of the curve $x = t^2/2$, $y = (2t+1)^{3/2}/3$, $0 \le t \le 4$. Express your answer as a definite integral; do NOT evaluate.

   We compute the derivatives $x' = t$ and $y' = (3/2)(2t+1)^{1/2}2/3 = (2t+1)^{1/2}$ so that the length of the curve is

$$\int_a^b \sqrt{(x')^2 + (y')^2}\,dt = \int_0^4 \sqrt{(t)^2 + ((1+2t)^{1/2})^2}\,dt = \int_0^\pi \sqrt{t^2+2t+1}\,dt$$

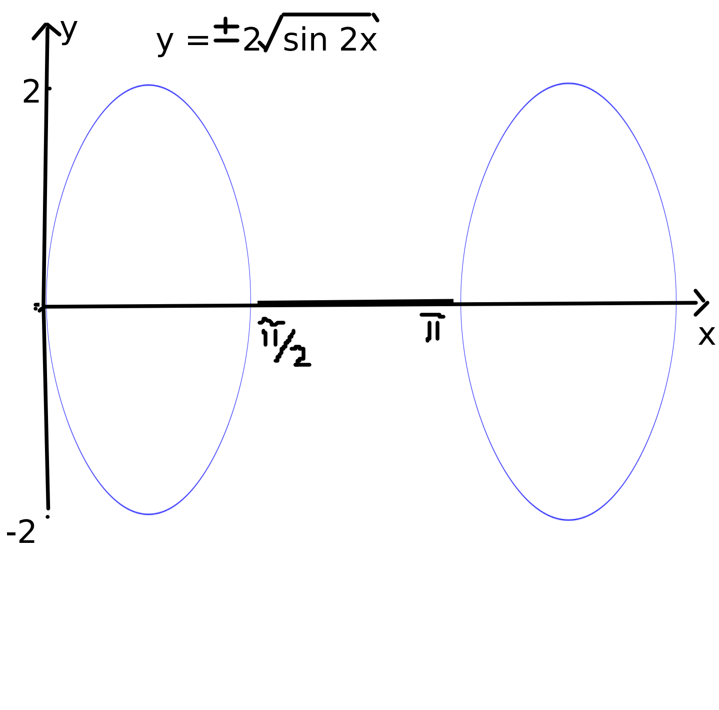
   (and this could be evaluated using the identity $t^2+2t+1 = (t+1)^2$ but that is not required here.)

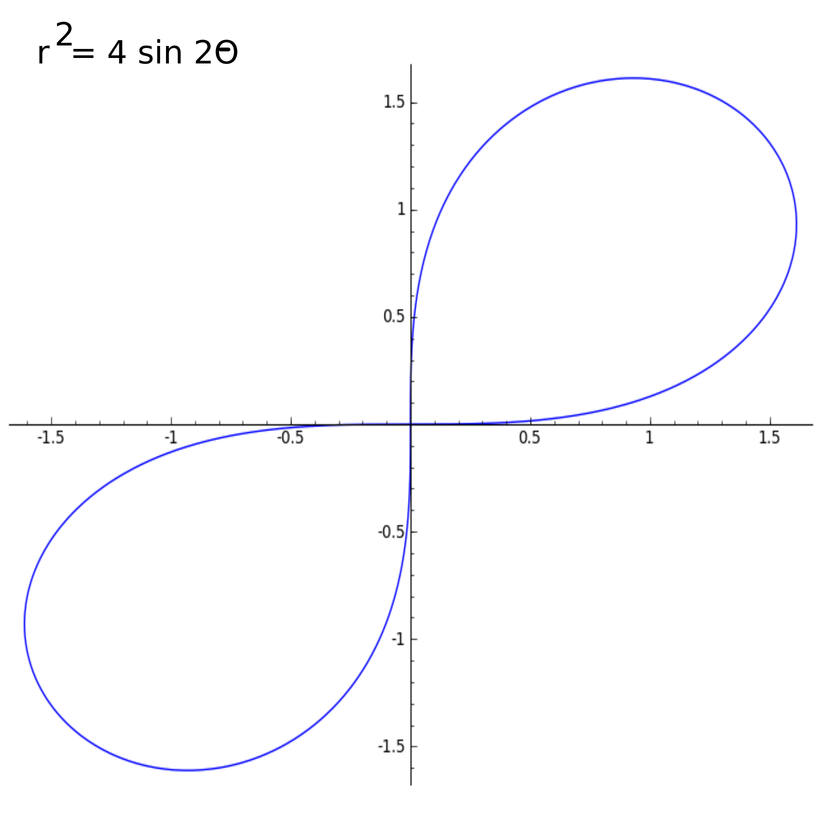
2. (4) Replace the polar equation $r = 8\sin\theta$ with an equivalent Cartesian equation. Then describe or identify the graph.

   Multiply by r: $r^2 = 8r\sin\theta$ and use the conversions $r^2 = x^2+y^2$ and $y = r\sin\theta$ so that we have $x^2+y^2 = 8y$. Subtract $8y$ from both sides and complete the square: $x^2+y^2-8y = 0$ or $x^2+y^2-8y+(-4)^2 = (-4)^2$ or $x^2+(y-4)^2 = 16$. This is a circle of radius $\sqrt{16} = 4$ and center (0,4).

3. (7) (a) Graph the leminiscate $r^2 = 4\sin 2\theta$

   Instead of creating a table of values, we graph $y^2 = 4\sin 2x$ or $y = \pm 2\sqrt{\sin 2x}$ and use that instead of the table of values. Observe that if $\sin 2x > 0$ there are two possible y values because of the $\pm$ and if $\sin 2x < 0$ then there are no possible values. Of course $\sin 2x > 0$ if $0 < x < \pi/2$ or $\pi < x < 3\pi/2$





(5) (b) Find the area inside one loop of the leminiscate $r^2 = 4\sin 2\theta$

By Part (a) above we see that one loop is traced out for θ in the interval $0 < \theta < \pi/2$ (or $\pi < x < 3\pi/2$) so that the area is

$$\int_0^{\pi/2} \frac{1}{2}(f(\theta))^2\, d\theta = \int_0^{\pi/2} \frac{1}{2} 4\sin 2\theta\, d\theta = 2\int_0^{\pi/2} \sin 2\theta\, d\theta = -\cos 2\theta|_0^{\pi/2} = -\cos\pi + \cos 0 = 2$$