Two Pages! **Quiz 9A**, Math 1860-021
4/17/14 Solutions Name ____________________

1. Find the length of the curve $x = \cos t$, $y = t + \sin t$, $0 \le t \le \pi$. Express your answer
(4) as a definite integral; do NOT evaluate.

We compute the derivatives $x' = -\sin t$ and $y' = 1 + \cos t$ so that the length of the curve is

$$\int_a^b \sqrt{(x')^2 + (y')^2}\,dt = \int_0^\pi \sqrt{(-\sin t)^2 + (1+\cos t)^2}\,dt = \int_0^\pi \sqrt{2 + 2\cos t}\,dt$$

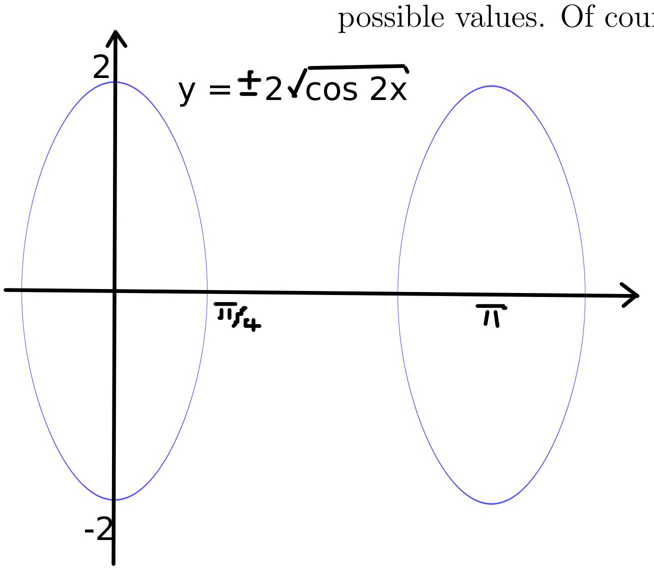
(and this could be evaluated using the identity $2 + 2\cos t = 4(\cos(t/2))^2$ but that is not required here.)

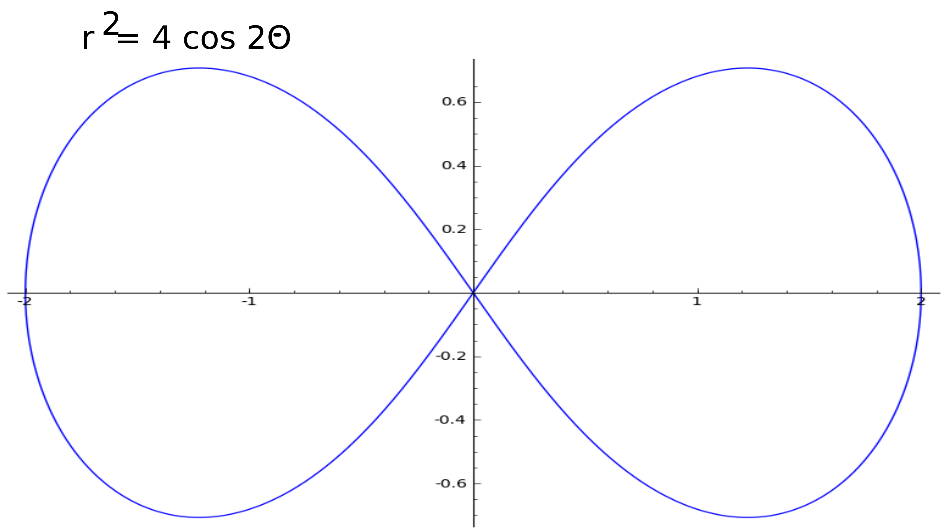
2. Replace the polar equation $r^2 = -4r\cos\theta$ with an equivalent Cartesian equation.
(4) Then describe or identify the graph.

Use the conversions $r^2 = x^2 + y^2$ and $x = r\cos\theta$ so that we have $x^2 + y^2 = -4x$. Add $4x$ to both sides and complete the square: $x^2 + y^2 + 4x = 0$ or $x^2 + 4x + (2)^2 + y^2 = (2)^2$ or $(x+2)^2 + y^2 = 4$. This is a circle of radius $\sqrt{4} = 2$ and center (-2,0).

(7) 3. (a) Graph the leminiscate $r^2 = 4\cos 2\theta$

Instead of creating a table of values, we graph $y^2 = 4\cos 2x$ or $y = \pm 2\sqrt{\cos 2x}$ and use that instead of the table of values. Observe that if $\cos 2x > 0$ there are two possible $y$ values because of the $\pm$ and if $\sin 2x < 0$ then there are no possible values. Of course $\cos 2x > 0$ if $-\pi/4 < x < \pi/4$ or $3\pi/4 < x < 5\pi/4$




(5) (b) Find the area inside one loop of the leminiscate $r^2 = 4\cos 2\theta$

By Part (a) above we see that one loop is traced out for $\theta$ in the interval $-\pi/4 < \theta < \pi/4$ (or $3\pi/4 < x < 5\pi/4$) so that the area is

$$\int_{-\pi/4}^{\pi/4} \frac{1}{2}(f(\theta))^2 \, d\theta = \int_{-\pi/4}^{\pi/4} \frac{1}{2} 4 \cos 2\theta \, d\theta = 2 \int_{-\pi/4}^{\pi/4} \cos 2\theta \, d\theta = \sin 2\theta|_{-\pi/4}^{\pi/4} = \sin \frac{\pi}{2} - \sin -\frac{\pi}{2} = 2$$