Two Pages! **Quiz 8B**, Math 1860-022
11/14/13 Solutions Name ____________________

1. Use power series operations to find the Taylor series at $x = 0$ for the function $\dfrac{x}{1-3x}$. (Suggestion: Recall the geometric series: $a/(1-r) = a + ar + ar^2 + ar^3 + \ldots$, for
(5) $-1 < r < 1$.)

   We apply the geometric series expansion with $a = x$ and $r = 3x$. Provided $-1 < 3x < 1$ (or $-1/3 < x < 1/3$)

   $$\frac{x}{1-3x} = x + x(3x) + x(3x)^2 + x(3x)^3 + x(3x)^4 + \ldots = x + 3x^2 + 3^2x^3 + 3^3x^4 + 3^4x^5 + \ldots$$

   (the nth term is $3^{n-1}x^n$, $n = 1, 2, 3, \ldots$.)

(6) 2. Find the Taylor polynomial of order 3 generated by $f(x) = \dfrac{1}{x}$ at $a = 2$

   Solution: We need to compute the first few derivatives evaluated at $x = a = 1$:

n	$f^{(n)}(x)$	$f^{(n)}(2)$
0	$1/x = x^{-1}$	$1/2$
1	$-x^{-2}$	$-1/4$
2	$2x^{-3}$	$2/8 = 1/4$
3	$-6x^{-4}$	$-3/8$

   Now substitute into the Taylor polynomial of degree 3.

   $$\begin{aligned} P_3(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 \\ &= \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{2!}\frac{1}{4}(x-2)^2 + \frac{1}{3!}\frac{-3}{8}(x-2)^3 \\ &= \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 \end{aligned}$$

3. Consider the curve with parametric equations $x = 4\sin t$ and $y = 2\cos t$, $0 \le t \le 2\pi$.

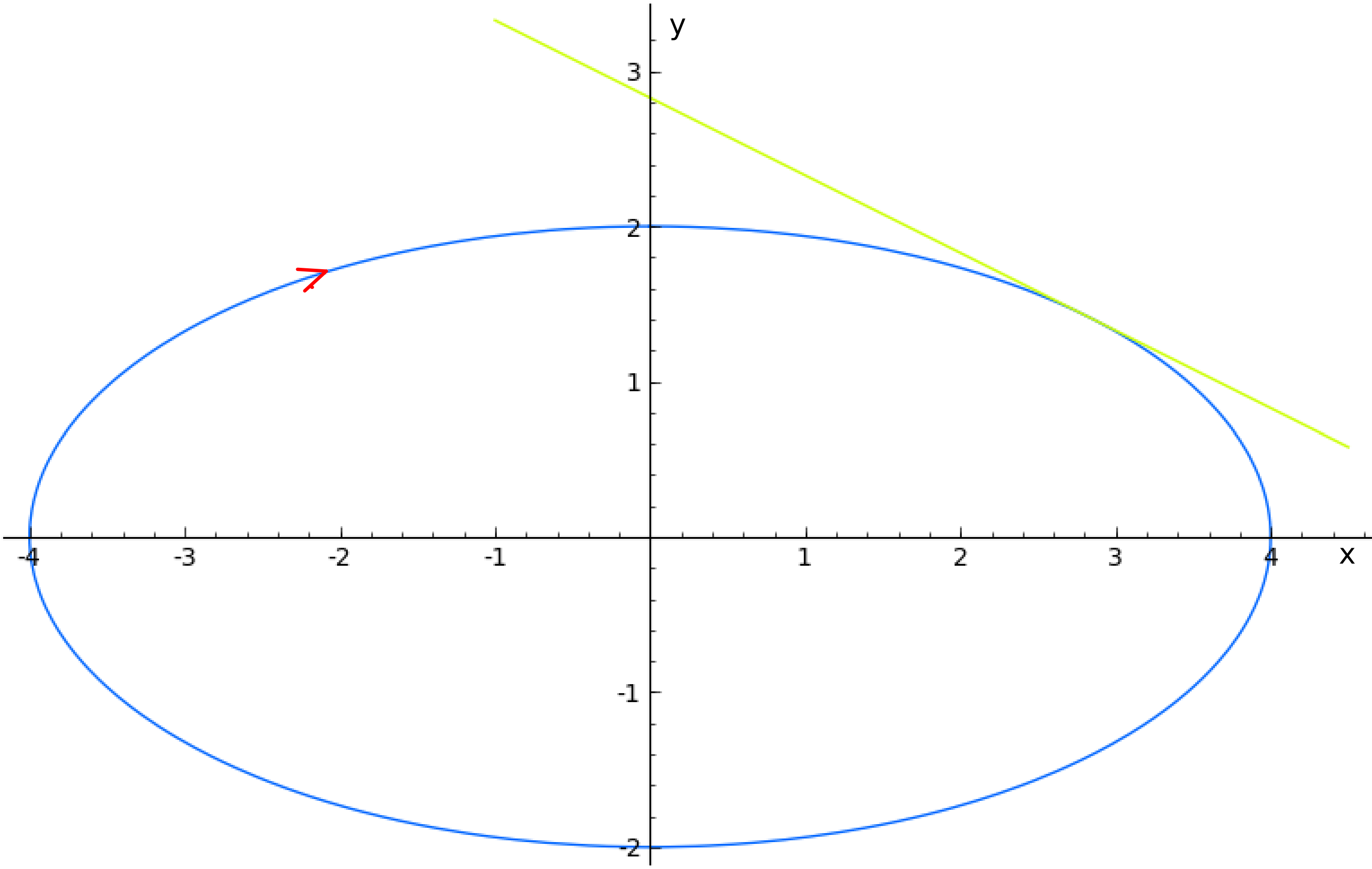
(9)

   (a) Identify the curve by finding a Cartesian equation for it. (Eliminate t.) Graph the curve and indicate which direction that the curve is traced out (as t increases).

   Eliminate t by noting $(x/4)^2 + (y/2)^2 = (\sin t)^2 + (\cos t)^2 = 1$ so that an equation for the curve is

   $$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

and this is an ellipse. When $t = 0$, $x = 0$ and $y = 2$; when $t = \pi/2$, $x = 4$ and



(b) Find an equation for the line tangent to the curve at the point given by $t = \pi/4$.
Find the slope

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin t}{4\cos t}$$

If we now substitute $t = \pi/4$ we find $dy/dx = -1/2$ and $y = \sqrt{2}$ and $x = 2\sqrt{2}$ and so the tangent line has equation

$$y - \sqrt{2} = -\frac{1}{2}(x - 2\sqrt{2})$$

or $y = -\frac{1}{2}x + 2\sqrt{2}$.