Two Pages! **Quiz 8A**, Math 1860-021
11/14/13 Solutions Name ____________________

1. Use power series operations to find the Taylor series at $x = 0$ for the function $\dfrac{x^2}{1-2x}$.
(5) (Suggestion: Recall the geometric series: $a/(1-r) = a + ar + ar^2 + ar^3 + \ldots$, for $-1 < r < 1$.)

We apply the geometric series expansion with $a = x^2$ and $r = 2x$. Provided $-1 < 2x < 1$ (or $-1/2 < x < 1/2$)

$$\frac{x^2}{1-2x} = x^2 + x^2(2x) + x^2(2x)^2 + x^2(2x)^3 + x^2(2x)^4 + \ldots = x^2 + 2x^3 + 2^2x^4 + 2^3x^5 + 2^4x^6 + \ldots$$

(the nth term is $2^{n-1}x^{n+1}$, $n = 1, 2, 3, \ldots$.)

(6) 2. Find the Taylor polynomial of order 3 generated by $f(x) = \dfrac{1}{x+2}$ at $a = 0$.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$1/(x+2) = (x+2)^{-1}$	$1/2$
1	$-(x+2)^{-2}$	$-1/4$
2	$2(x+2)^{-3}$	$2/8 = 1/4$
3	$-6(x+2)^{-4}$	$-3/8$

Now substitute into the formula for the Taylor polynomial of degree 3.

$$\begin{aligned} T_3(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 \\ &= \frac{1}{2} - \frac{1}{4}x + \frac{1}{2!}\frac{1}{4}x^2 + \frac{1}{3!}\frac{-3}{8}x^3 \\ &= \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 \end{aligned}$$

3. Consider the curve with parametric equations $x = 5\cos t$ and $y = 4\sin t$, $0 \le t \le 2\pi$.

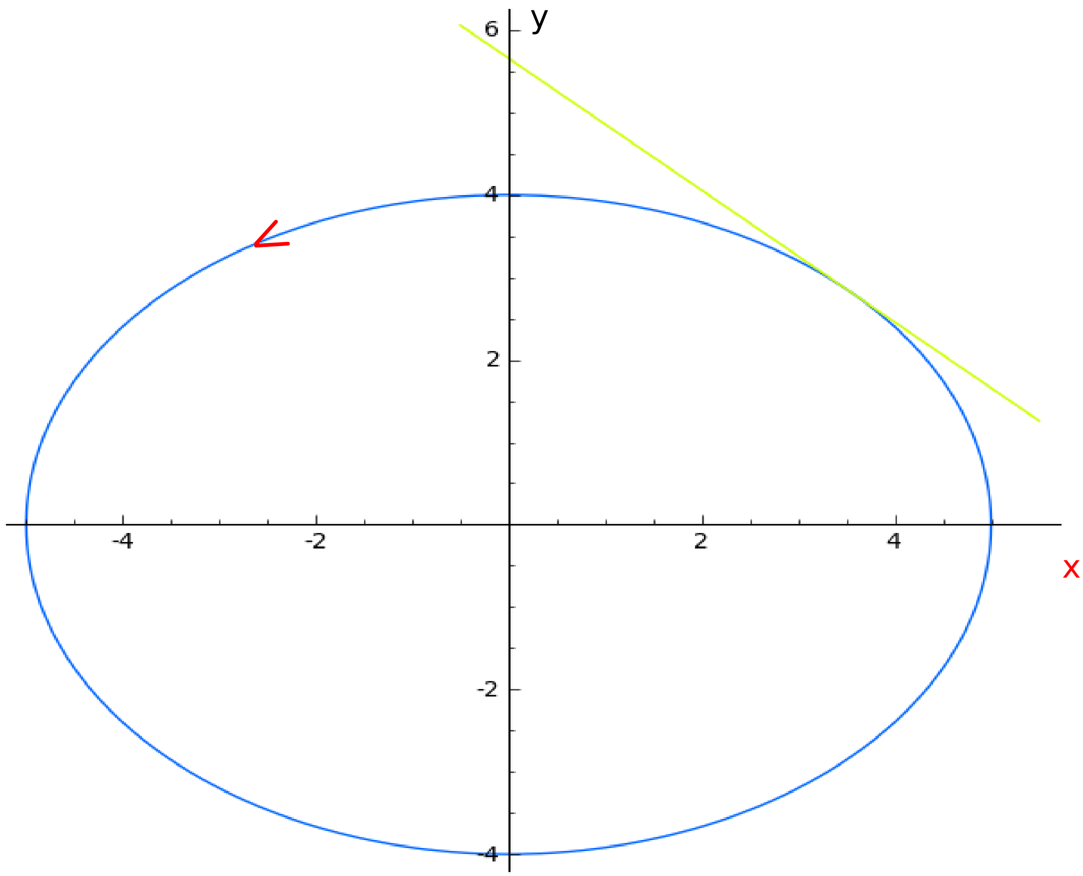
(9)

(a) Identify the curve by finding a Cartesian equation for it. (Eliminate t.) Graph the curve and indicate which direction that the curve is traced out (as t increases).

Eliminate t by noting $(x/5)^2 + (y/4)^2 = (\cos t)^2 + (\sin t)^2 = 1$ so that an equation for the curve is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

and this is an ellipse. When $t = 0$, $x = 5$ and $y = 0$; when $t = \pi/2$, $x = 0$ and $y = 4$ and when $t = 2\pi$, $x = 5$ and $y = 0$ and so by plotting a few points like these we see the curve is traced out counterclockwise and the full ellipse is traced out.



(b) Find an equation for the line tangent to the curve at the point given by $t = \pi/4$.

Find the slope

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-5\sin t}$$

If we now substitute $t = \pi/4$ we find $dy/dx = -4/5$ and $y = 2\sqrt{2}$ and $x = 5\sqrt{2}/2$ and so the tangent line has equation

$$y - 2\sqrt{2} = -\frac{4}{5}(x - 5\sqrt{2}/2)$$

or $y = -4x/5 + 4\sqrt{2}$.