Two Pages! **Quiz 7A**, Math 1860-021
3/20/14 Solutions Name ____________________

(4) 1. Use the Comparison Test to determine if the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$$

We want to compare to $\sum_{n=1}^{\infty} \frac{1}{n}$ which is a p series with $p=1$ (harmonic series) and since $p \le 1$ it diverges. Try the Comparison Test

$$\frac{n+2}{n^2-n} \ge \frac{n}{n^2-n} \ge \frac{n}{n^2} = \frac{1}{n}$$

and we know that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and so the given series diverges.

(4) 2. Use the Limit Comparison Test to determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \sqrt{\frac{n-2}{n^3-n^2+3}}$$

We want to compare to $\sum_{n=1}^{\infty} \frac{1}{n}$ which is a p series with $p=1$ and since $p \le 1$ it diverges. Try Limit Comparison

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{\dfrac{n-2}{n^3-n^2+3}}}{\dfrac{1}{n}} = \lim_{n\to\infty} \sqrt{n^2}\sqrt{\frac{(n-2)}{n^3-n^2+3}} = \sqrt{\lim_{n\to\infty} \frac{n^2(n-2)}{n^3-n^2+3}} = 1$$

where the limit follows by l'Hôpital's Rule (or factor n^3 out of top and bottom). Since the limit is $0 < 1 < \infty$, the Limit Comparison Test says that the two series both converge or both diverge but we know that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and so the given series diverges.

(4) 3. Does the series converge or diverge? Use any method but give reasons for your answer.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}$$

We compare to $\sum_{n=2}^{\infty} \frac{1}{n^2}$ which is a p series with $p=2>1$ and so it converges. Limit Comparison applies here.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\dfrac{1}{n\sqrt{n^2-1}}}{1/n^2} = \lim_{n\to\infty} \frac{n^2}{n\sqrt{n^2-1}} = \lim_{n\to\infty} \frac{n}{(\sqrt{1-1/n^2})n} = 1$$

And this limit is $0 < 1 < \infty$ and so the series both converge or diverge but $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges and so the given series must also converge.

(4 ea) 4. Does the *series* converge or diverge? Give reasons for your answer. The ratio or root test might help

(a) $\sum_{n=1}^{\infty} \frac{4^n}{n!}$

This is a good candidate for the ratio test because there are lots of multiplications between terms. Consider therefore

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{4^{n+1}}{(n+1)!}}{\frac{4^n}{n!}} = \lim_{n\to\infty} \frac{4^{n+1}}{4^n} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{4}{n+1} = 0$$

(where we note $4^{n+1} = 4(4^n)$ and $(n+1)! = (n+1)n!$.) Since the limit is smaller than 1 the series converges absolutely by the ratio test.

(b) $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{(n)^n}$

This is a good candidate for the root test because there are powers of n. Consider therefore

$$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \frac{\ln n}{n} \left(= \frac{\infty}{\infty}\right) = \lim_{n\to\infty} \frac{1/n}{1} = 0$$

by l'Hôpital's rule. The limit is smaller than 1 and so the series converges absolutely by the root test.