Two Pages! **Quiz 4B**, Math 1860-022
9/19/13 Solutions Name ________________

1. Evaluate the integrals in Parts (a) to (d). Use integration by parts as appropriate.

(5 ea)

(a) $\int \theta \cos \pi\theta \, d\theta$

We integrate by parts setting $u = \theta$ and $dv = \cos \pi\theta \, d\theta$. Therefore $du = d\theta$ and $v = (1/\pi)\sin(\pi\theta)$. We recall the formula $\int u\,dv = uv - \int v\,du$. Therefore

$$\int \theta \cos \pi\theta \, d\theta = \frac{\theta}{\pi}\sin(\pi\theta) - \int \frac{1}{\pi}\sin(\pi\theta)\,d\theta = \frac{\theta}{\pi}\sin(\pi\theta) + \frac{1}{\pi^2}\cos(\pi\theta)$$

Check by differentiation:

$$\frac{d}{dx}\left[\frac{\theta}{\pi}\sin(\pi\theta) + \frac{1}{\pi^2}\cos(\pi\theta)\right] = \frac{\theta}{\pi}\cos(\pi\theta)\pi + \frac{1}{\pi}\sin(\pi\theta) + \frac{1}{\pi^2}(-\sin(\pi\theta)\pi) = \theta\cos(\pi\theta)$$

(b) $\int \tan^{-1} y \, dy$

We integrate by parts setting $u = \tan^{-1} y$ and $dv = dx$ so that $du = 1/(1 + y^2)\,dy$ and $v = y$. Therefore

$$\int \tan^{-1} y\,dy = y\tan^{-1} y - \int \frac{y}{1+y^2}\,dy$$

To evaluate the integral we use u-substitution: $u = 1 + y^2$ so that $du = 2y\,dy$

$$\begin{aligned}\int \tan^{-1} y\,dy &= y\tan^{-1} y - \frac{1}{2}\int \frac{1}{u}\,du \\ &= y\tan^{-1} y - \frac{1}{2}\ln|u| + C = y\tan^{-1} y - \frac{1}{2}\ln(1+y^2) + C\end{aligned}$$

Check by differentiation.

$$\frac{d}{dy}\left[y\tan^{-1} y - \frac{1}{2}\ln(1+y^2)\right] = y\,\frac{1}{1+y^2} + \tan^{-1} y - \frac{1}{2}\frac{1}{1+y^2}2y = \tan^{-1} y$$

(c) $\int \cos^3 x\,dx$

Substitute $u = \sin x$ so that $du = \cos x$ and $(\cos x)^2 = 1 - (\sin x)^2 = 1 - u^2$. Therefore

$$\int \cos^3 x\,dx = \int 1 - u^2\,du = \left[u - \frac{1}{3}u^3\right] + C = \sin x - \frac{1}{3}(\sin x)^3 + C$$

We can check by differentiation.

$$\begin{aligned}\frac{d}{dx}\left[\sin x - \frac{1}{3}(\sin x)^3\right] &= \cos x - \frac{1}{3}3(\sin x)^2(\cos x) \\ &= \cos x\left[1 - (\sin x)^2\right] \\ &= \cos x\left[\cos^2 x\right] = (\cos x)^3\end{aligned}$$

(d) $\int 16\sin^2 x \cos^2 x \, dx$

Use the identities $\sin^2 x = (1/2)(1 - \cos 2x)$ and $\cos^2 x = (1/2)(1 + \cos 2x)$. The integrand is therefore

$$16\sin^2 x \cos^2 x = 4(1 - \cos 2x)(1 + \cos 2x) = 4(1 - (\cos 2x)^2) = 4(\sin 2x)^2$$

and again we apply the identity $(\sin 2x)^2 = (1/2)(1 - \cos 4x)$. Therefore $16\sin^2 x \cos^2 x = 2 - 2\cos 4x$.

(An alternative approach is to use the identity $2\sin x \cos x = \sin 2x$ so that $16\sin^2 x \cos^2 x = 4(\sin 2x)^2 = 2 - 2\cos 4x$.)

We are ready to integrate

$$\int 16\sin^2 x \cos^2 x \, dx = \int 2 - 2\cos 4x \, dx = 2x - \frac{1}{2}\sin 4x + C.$$

The check involves differentiation and reversing the use of identities above.