Two Pages! **Quiz 4A**, Math 1860-021

9/19/13 Solutions Name ____________________

1. Evaluate the integrals in Parts (a) to (d). Use integration by parts as appropriate.

(5 ea)

(a) $\int xe^{3x}\,dx$

We integrate by parts setting $u = x$ and $dv = e^{3x}\,dx$. Therefore $du = dx$ and $v = (1/3)e^{3x}$. We recall the formula $\int u\,dv = uv - \int v\,du$. Therefore

$$\int xe^{3x}\,dx = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

Check by differentiation: By the product rule

$$\frac{d}{dx}\left[\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right] = \frac{1}{3}xe^{x/3}\,3 + \frac{1}{3}e^{3x} - \frac{1}{9}e^{3x}\,3 = xe^{x/3}$$

(b) $\int \tan^{-1} y\,dy$

We integrate by parts setting $u = \tan^{-1} y$ and $dv = dx$ so that $du = 1/(1+y^2)\,dy$ and $v = y$. Therefore

$$\int \tan^{-1} y\,dy = y\tan^{-1} y - \int \frac{y}{1+y^2}\,dy$$

To evaluate the integral we use u-substitution: $u = 1 + y^2$ so that $du = 2y\,dy$

$$\begin{aligned}\int \tan^{-1} y\,dy &= y\tan^{-1} y - \frac{1}{2}\int \frac{1}{u}\,du \\ &= y\tan^{-1} y - \frac{1}{2}\ln|u| + C = y\tan^{-1} y - \frac{1}{2}\ln(1+y^2) + C\end{aligned}$$

Check by differentiation.

$$\frac{d}{dy}\left[y\tan^{-1} y - \frac{1}{2}\ln(1+y^2)\right] = y\,\frac{1}{1+y^2} + \tan^{-1} y - \frac{1}{2}\frac{1}{1+y^2}2y = \tan^{-1} y$$

(c) $\int \sin^3 x\,dx$

Substitute $u = \cos x$ so that $du = -\sin x$ and $(\sin x)^2 = 1 - (\cos x)^2 = 1 - u^2$. Therefore

$$\int \cos^3 x\,dx = -\int 1 - u^2\,du = -\left[u - \frac{1}{3}u^3\right] + C = -\cos x + \frac{1}{3}(\cos x)^3 + C$$

We can check by differentiation.

$$\begin{aligned}\frac{d}{dx}\left[-\cos x + \frac{1}{3}(\cos x)^3\right] &= \sin x + \frac{1}{3}3(\cos x)^2(-\sin x) \\ &= \sin x\left[1 - (\cos x)^2\right] \\ &= \sin x\left[\sin^2 x\right] = (\sin x)^3\end{aligned}$$

(d) $\int 16\sin^2 x\cos^2 x\,dx$

Use the identities $\sin^2 x = (1/2)(1 - \cos 2x)$ and $\cos^2 x = (1/2)(1 + \cos 2x)$. The integrand is therefore

$$16\sin^2 x\cos^2 x = 4(1 - \cos 2x)(1 + \cos 2x) = 4(1 - (\cos 2x)^2) = 4(\sin 2x)^2$$

and again we apply the identity $(\sin 2x)^2 = (1/2)(1 - \cos 4x)$. Therefore $16\sin^2 x\cos^2 x = 2 - 2\cos 4x$.

(An alternative approach is to use the identity $2\sin x\cos x = \sin 2x$ so that $16\sin^2 x\cos^2 x = 4(\sin 2x)^2 = 2 - 2\cos 4x$.)

We are ready to integrate

$$\int 16\sin^2 x\cos^2 x\,dx = \int 2 - 2\cos 4x\,dx = 2x - \frac{1}{2}\sin 4x + C.$$

The check involves differentiation and reversing the use of identities above.