Two Pages! **Quiz 3B**, Math 1860-022

1/30/14 Solutions Name ____________________

(8) 1. Find the area of the surface generated by revolving the curve $x = y^3/3$, $0 \le y \le 1$ about the y–axis.

We need $dx/dy = y^2$ so that the surface area is

$$\int_a^b 2\pi x\sqrt{1+(dx/dy)^2}\,dy = 2\pi\int_0^1 \frac{y^3}{3}\sqrt{1+(y^2)^2}\,dy = 2\pi\int_0^1 \frac{y^3}{3}\sqrt{1+y^4}\,dy$$

Substitute $u = 1 + y^4$ so that $du = 4y^3\,dy$ or $(1/4)\,du = y^3\,dy$. Therefore the surface area is

$$2\pi\int_0^1 \frac{y^3}{3}\sqrt{1+y^4}\,dy = \frac{2\pi}{12}\int_{y=0}^{y=2}\sqrt{u}\,du = \frac{\pi}{6}\frac{2}{3}u^{3/2}|_{y=0}^{y=1} = \frac{\pi}{9}(1+y^4)^{3/2}|_0^1 = \frac{\pi}{9}\left[(2)^{3/2}-1\right]$$

(4 ea) 2. Evaluate the integrals.

(a) $\displaystyle\int \frac{2y\,dy}{y^2-25}$

Substitute $u = y^2 - 25$ so that $du = 2y\,dx$. We can change the limits of so that

$$\int \frac{2y\,dy}{y^2-25} = \int \frac{1}{u}\,du = \ln|u| + C = \ln|y^2-25| + C$$

(b) $\displaystyle\int_1^4 \frac{(\ln x)^3}{2x}\,dx$

Substitute $u = \ln x$ so that $du = (1/x)\,dx$. Therefore

$$\int_1^4 \frac{(\ln x)^3}{2x}\,dx = \frac{1}{2}\int_{x=1}^{x=4} u^3\,du = \frac{1}{8}u^4|_{x=1}^{x=4} = \frac{1}{8}(\ln x)^4|_1^4 = \frac{1}{8}\left[(\ln 4)^4 - (\ln 1)^4\right] = \frac{1}{8}(2\ln 2)^4 = 2(\ln 2)^4$$

(c) $\displaystyle\int_0^{\pi/2} 7^{\cos t}\sin t\,dt$

Substitute $u = \cos t$ so that $du = -\sin t\,dt$. When $t = 0$, $u = 1$ and when $t = \pi/2$, $u = 0$ so that

$$\int_0^{\pi/2} 7^{\cos t}\sin t\,dt = -\int_1^0 7^u\,du = -\frac{1}{\ln 7}7^u|_1^0 = -\frac{1}{\ln 7}[7^0 - 7^1] = \frac{6}{\ln 7}$$