Two Pages! **Quiz 3A**, Math 1860-021
1/30/14 Solutions Name ____________________

1. Find the area of the surface generated by revolving the curve $y = x^3/9$, $0 \leq x \leq 2$
(8) about the x–axis.

We need $dy/dx = x^2/3$ so that the surface area is

$$\begin{aligned}\int_a^b 2\pi y\sqrt{1+(dy/dx)^2}\,dx &= 2\pi\int_0^2 \frac{x^3}{9}\sqrt{1+(x^2/3)^2}\,dx \\ &= 2\pi\int_0^2 \frac{x^3}{9}\sqrt{1+x^4/9}\,dx\end{aligned}$$

Substitute $u = 1 + x^4/9$ so that $du = 4x^3/9\,dx$ or $(1/4)\,du = x^3/9\,dx$. Therefore the surface area is

$$\begin{aligned}2\pi\int_0^2 \frac{x^3}{9}\sqrt{1+x^4/9}\,dx &= \frac{2\pi}{4}\int_{x=0}^{x=2}\sqrt{u}\,du \\ &= \frac{\pi}{2}\frac{2}{3}u^{3/2}|_{x=0}^{x=2} = \frac{\pi}{3}(1+x^4/9)^{3/2}|_0^2 = \frac{\pi}{3}\left[\left(\frac{25}{9}\right)^{3/2} - 1\right] = \frac{98\pi}{81}\end{aligned}$$

(4 ea) 2. Evaluate the integrals.

(a) $\displaystyle\int_{-1}^0 \frac{3\,dx}{3x-2}$

Substitute $u = 3x - 2$ so that $du = 3\,dx$. We can change the limits of integration: when $x = -1$, $u = -5$ and when $x = 0$, $u = -2$ so that

$$\int_{-1}^0 \frac{3\,dx}{3x-2} = \int_{-5}^{-2}\frac{1}{u}\,du = \ln|u||_{-5}^{-2} = \ln|-2| - \ln|-5| = \ln 2 - \ln 5 = \ln(2/5)$$

(b) $\displaystyle\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\,dr$

Substitute $u = \sqrt{r} = r^{1/2}$ so that $du = (1/2)r^{-1/2}\,dr$ or $2du = (1/\sqrt{r})\,dr$. Therefore

$$\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\,dr = 2\int e^u\,du = 2e^u = 2e^{r^{1/2}} + C$$

Check by differentiation.

$$\frac{d}{dr}2e^{r^{1/2}} = 2e^{r^{1/2}}\frac{1}{2}r^{-1/2} = \frac{e^{\sqrt{r}}}{\sqrt{r}}$$

It checks

(c) $\int_0^{\pi/2} 7^{\cos t} \sin t \, dt$

Substitute $u = \cos t$ so that $du = -\sin t \, dt$. When $t = 0$, $u = 1$ and when $t = \pi/2$, $u = 0$ so that

$$\int_0^{\pi/2} 7^{\cos t} \sin t \, dt = -\int_1^0 7^u \, du = -\frac{1}{\ln 7} 7^u \big|_1^0 = -\frac{1}{\ln 7}[7^0 - 7^1] = \frac{6}{\ln 7}$$