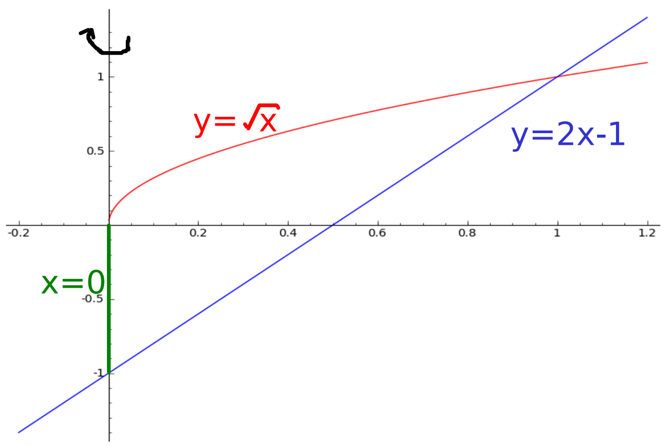
Two Pages! **Quiz 2A**, Math 1860-022
1/23/14 Solutions Name ____________________

1. Use the shell method to find the volume of the solid generated by revolving the
(7) region bounded by the curves $y = 2x - 1$, $y = \sqrt{x}$, and $x = 0$ about the y–axis.

   Sketch the region. The curves $y = 2x - 1$, $y = \sqrt{x}$ intersect when $\sqrt{x} = 2x - 1$ or $0 = 2(\sqrt{x})^2 - \sqrt{x} - 1 = (2\sqrt{x} + 1)(\sqrt{x} - 1)$ so that $\sqrt{x} = 1$ which means $x = 1$. The volume is

$$\begin{aligned}\int_0^1 2\pi x(\sqrt{x} - (2x-1))\,dx &= 2\pi \int_0^1 x^{3/2} - 2x^2 + x\,dx \\ &= 2\pi \left[\frac{2}{5}x^{5/2} - \frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^1 \\ &= 2\pi \left[(\frac{2}{5}1^{5/2} - \frac{2}{3}1^3 + \frac{1}{2}1^2) - 0\right] = \frac{7\pi}{15}\end{aligned}$$



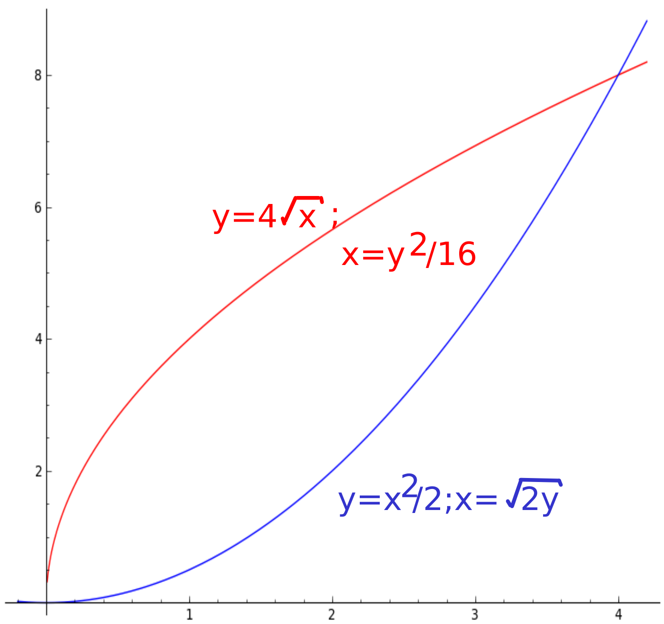
2. Find the volume of the solid generated when the region bounded by the curves $y = 4\sqrt{x}$ and $y = x^2/2$ is rotated (a) about the x–axis and (b) about the y–axis. In each case express your answer as a definite integral, ready to be evaluated, but
(4 ea) do NOT evaluate. (Suggestion: Sketch the region.)

   (a) The volume (as an integral) if the region is rotated about the x-axis
   Sketch the region. The curves intersect when $4\sqrt{x} = x^2/2$ or $8\sqrt{x} - x^2 = 0$ or $\sqrt{x}(8 - x^{3/2})$ so that $x = 0$ or $x = 8^{2/3} = 4$. Using the washer method we get the volume V is

$$V = \int_0^4 \pi(4\sqrt{x})^2 - \pi(x^2/2)^2\,dx = \pi \int_0^4 16x - x^4/4\,dx$$

   Alternatively cylindrical shells could be used but we need the bounding curves as functions of y: $x = y^2/16$ and $x = \sqrt{2y}$ which intersect when $y = 0$ and $y = 8$. The volume is

$$V = \int_0^8 2\pi y(\sqrt{2y} - y^2/16)\,dy = 2\pi \int_0^4 \sqrt{2}y^{3/2} - y^3/16\,dy$$



(b) The volume (as an integral) if the region is rotated about the y-axis
This is a different solid from Part 1 but this problem has a lot of symmetries. If we use cylindrical shells we get

$$V = \int_0^4 2\pi x \left[4\sqrt{x} - x^2/2\right]\, dx = 2\pi \int_0^4 4x^{3/2} - x^3/2\, dx$$

or we can use washers.

$$V = \int_0^8 \pi(\sqrt{2y})^2 - \pi(y^2/16)^2\, dy = \pi \int_0^4 2y - y^4/256\, dy$$

(5) 3. Set up an integral for the length of the curve $y = \tan x$, $-\pi/3 \leq x \leq 0$. Do NOT evaluate.

We need $y' = (\sec x)^2$. The length of the curve is given by the integral

$$\int_{-\pi/3}^{0} \sqrt{1 + ((\sec x)^2)^2}\, dx = \int_{-\pi/3}^{0} \sqrt{1 + (\sec x)^4}\, dx$$