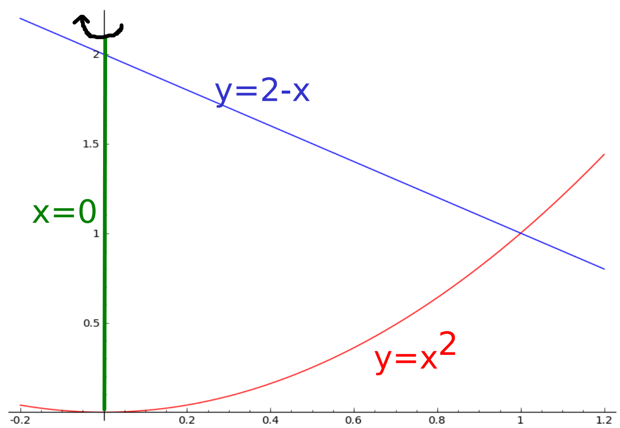
Two Pages! **Quiz 2A**, Math 1860-021

1/23/14 Solutions Name ____________________

1. Use the shell method to find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and $y = 2 - x$, $x = 0$, for $x \geq 0$ about the
(7) y–axis.

Graph the region. The two curves $y = x^2$ and $y = 2 - x$ intersect when $x^2 = 2 - x$ or $x^2 + x - 2 = 0$ or $(x-1)(x+2) = 0$ so that $x = 1$ or $x = -2$ but we have $x \geq 0$ and so $x = 1$. By cylindrical shells, the volume is

$$\begin{aligned} V &= \int_0^1 2\pi x(2 - x - x^2)\,dx = 2\pi \int_0^1 2x - x^2 - x^3\,dx \\ &= 2\pi \left[x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = 2\pi[1 - \frac{1}{3} - \frac{1}{4} - 0] = \frac{5\pi}{6} \end{aligned}$$



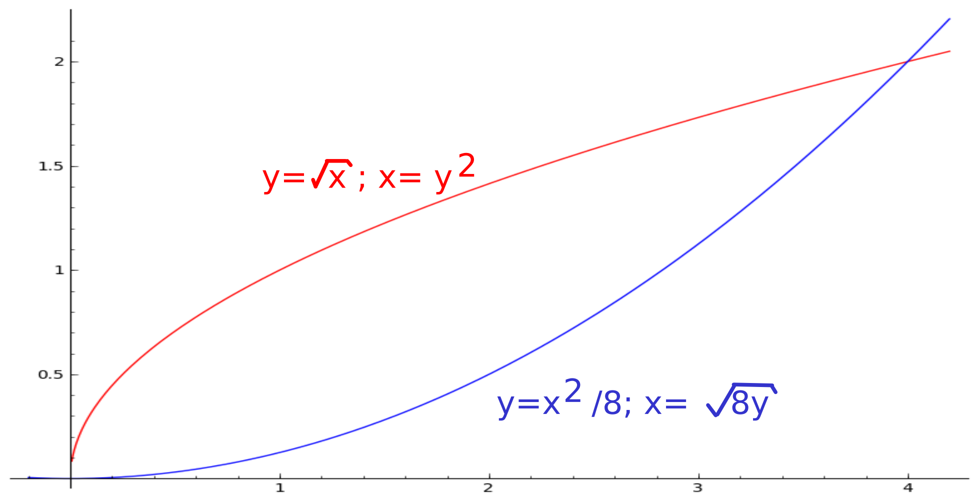
2. Find the volume of the solid generated when the region bounded by the curves $y = \sqrt{x}$ and $y = x^2/8$ is rotated (a) about the x axis and (b) about the y-axis. In each case express your answer as a definite integral, ready to be evaluated, but do
(4 ea) NOT evaluate. (Suggestion: Sketch the region.)

   (a) The volume (as an integral) if the region is rotated about the x-axis
   Sketch the region. The curves intersect when $\sqrt{x} = x^2/8$ or $8\sqrt{x} - x^2 = 0$ or $\sqrt{x}(8 - x^{3/2})$ so that $x = 0$ or $x = 8^{2/3} = 4$. Using the washer method we get the volume V is

$$V = \int_0^4 \pi(\sqrt{x})^2 - \pi(x^2/8)^2\,dx = \pi \int_0^4 x - x^4/64\,dx$$

   Alternatively cylindrical shells could be used but we need the bounding curves as functions of y: $x = y^2$ and $x = \sqrt{8y}$ which intersect when $y = 0$ and $y = 2$. The volume is

$$V = \int_0^2 2\pi y(\sqrt{8y} - y^2)\,dy = 2\pi \int_0^2 2^{3/2}y^{3/2} - y^3/4\,dy$$



(b) The volume (as an integral) if the region is rotated about the y-axis

This is a different solid from Part 1. If we use cylindrical shells we get

$$V = \int_0^4 2\pi x \left[\sqrt{x} - x^2/8\right] \, dx = 2\pi \int_0^4 x^{3/2} - x^3/8 \, dx$$

or we can use washers.

$$V = \int_0^2 \pi(\sqrt{8y})^2 - \pi(y^2)^2 \, dy = \pi \int_0^2 8y - y^4 \, dy$$

(5) 3. Set up an integral for the length of the curve $x = \sin y$, $0 \leq y \leq \pi$. Do NOT evaluate.

We see that $x' = \cos y$ so that the lenght of the curve is

$$\int_0^\pi \sqrt{1 + (\cos y)^2} \, dy$$