Two Pages! **Quiz 1B**, Math 1860-003
1/19/12 Solutions Name ____________________

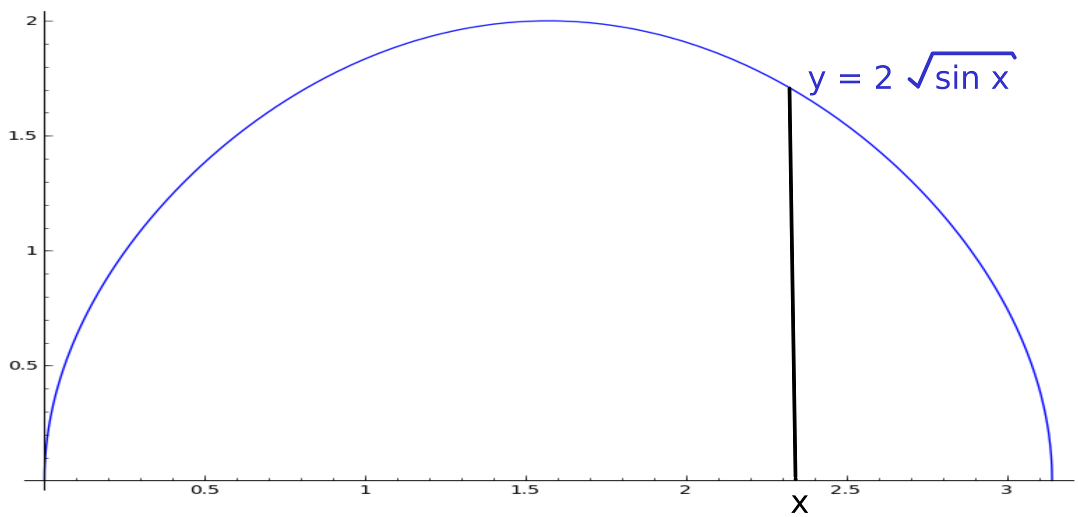
1. Find the volume of the solid whose base is the region between the curve $y = 2\sqrt{\sin x}$ amd the interval $[0, \pi]$ on the $x$-axis. The cross sections perpendicular to the $x$-axis are squares whose bases run from the $x$-axis to the curve. (10)

Sketch the solid or at least the base of the solid. The volume is

$$V = \int_a^b A(x)\, dx = \int_0^\pi A(x)\, dx$$

where $A(x)$ is the area of the cross section which is a square with base length $2\sqrt{\sin x}$. Therefore the cross sectional area is $A(x) = (2\sqrt{\sin x})^2 = 4 \sin x$ at $x$ units from the $y$-axis, $0 \le x \le \pi$. Therefore the volume is

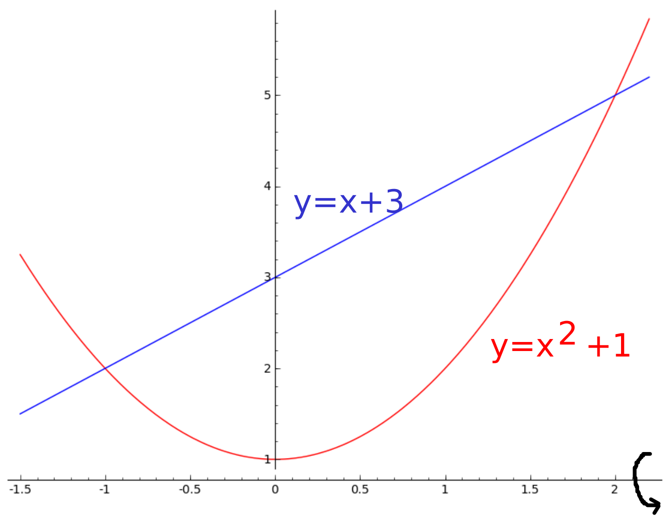
$$V = \int_0^\pi 4 \sin x\, dx = -4 \cos x|_0^\pi = -4 \cos \pi - (-4 \cos 0) = 8$$




2. Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2 + 1$ and $y = x + 3$ about the $x$-axis. (10)

Sketch the region. The curves intersect when $x^2 + 1 = x + 3$ or $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$ which means $x = -1$ or $x = 2$. The region is bounded above and below by functions of $x$ and so is of "Type I" and the method of washers applies. The volume is

$$\begin{aligned}
V &= \int_{-1}^2 \pi(x+3)^2 - \pi(x^2+1)^2\, dx \\
&= \pi \int_{-1}^2 (x^2 + 6x + 9) - (x^4 + 2x^2 + 1)\, dx \\
&= \pi \int_{-1}^2 -x^4 - x^2 + 6x + 8\, dx \\
&= \pi \left[ -\frac{1}{5} x^5 - \frac{1}{3} x^3 + 3x^2 + 8x \right]_{-1}^2 \\
&= \pi \left[ -\frac{1}{5} 32 - \frac{8}{3} + 12 + 16 - (-\frac{1}{5}(-1)^5 + \frac{1}{3} + 3 - 8) \right] = \frac{117\pi}{5}
\end{aligned}$$


y=x+3
y=x^2+1
5
4
3
2
1
-1.5
-1
-0.5
0
0.5
1
1.5
2