Two Pages! **Quiz 1A**, Math 1860-021
1/16/14 Solutions Name ____________________

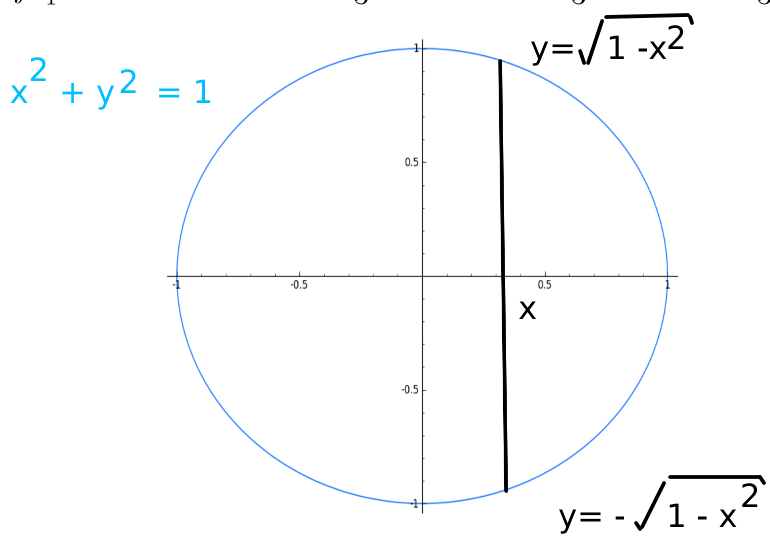
1. Find the volume of the solid which lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis between these planes are squares whose bases run from the semicircle $y = -\sqrt{1-x^2}$ to the
(10) semicircle $y = \sqrt{1-x^2}$.

   Sketch the solid or at least the base of the solid which is a circle of radius 1 because if $y = \pm\sqrt{1-x^2}$ then $x^2 + y^2 = 1$. The volume is

$$V = \int_a^b A(x)\,dx = \int_{-1}^1 A(x)\,dx$$

   where $A(x)$ is the area of the cross section which is a square with base length $\sqrt{1-x^2} - (-\sqrt{1-x^2}) = 2\sqrt{1-x^2}$. Therefore the cross sectional area is $A(x) = (2\sqrt{1-x^2})^2 = 4(1-x^2)$ at x units from the y-axis, $-1 \le x \le 1$. Therefore the volume is

$$V = \int_{-1}^1 4(1-x^2)\,dx = 4[x - \frac{1}{3}x^3|_{-1}^1 = 4[1 - \frac{1}{3}1^3 - (-1 - \frac{1}{3}(-1)^3)] = \frac{16}{3}$$



2. Find the volume of the solid generated by revolving the region bounded by the
(10) curve $y = x^2 + 2$ and $y = x + 4$ about the x-axis.

   Sketch the region. The curves intersect when $x^2 + 2 = x + 4$ or $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$ which means $x = -1$ or $x = 2$. The region is bounded above and below by functions of x and so is of "Type I" and the method of washers applies. The volume is

$$\begin{aligned} V &= \int_{-1}^2 \pi(x+4)^2 - \pi(x^2+2)^2\,dx \\ &= \pi\int_{-1}^2 (x^2+8x+16) - (x^4+4x^2+4)\,dx \\ &= \pi\int_{-1}^2 -x^4 - 3x^2 + 8x + 12\,dx \end{aligned}$$

$$= \pi\left[-\frac{1}{5}x^5 - x^3 + 4x^2 + 12x\right]_{-1}^{2}$$
$$= \pi\left[-\frac{1}{5}32 - 8 + 16 + 24 - (-\frac{1}{5}(-1)^5 + 1 + 4 - 12)\right] = \frac{162\pi}{5}$$