Formula Sheet,

5-2 -07 Math 1860 Final Exam

Remember: Your formula sheet must be handwritten!

Some Integrals:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\, dx = \ln|\csc x - \cot x| + C$$

Trigonometry:

$$(\sin x)^2 + (\cos x)^2 = 1 \qquad (\tan x)^2 + 1 = (\sec x)^2 \qquad (\cot x)^2 + 1 = (\csc x)^2$$

$$(\sin x)^2 = \frac{1}{2}(1-\cos 2x) \qquad (\cos x)^2 = \frac{1}{2}(1+\cos 2x)$$

$$\sin 2x = 2\sin x\cos x, \qquad \cos 2x = (\cos x)^2 - (\sin x)^2$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)] \qquad \cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

Parametric Curves: $x(t)$, $y(t)$. The arc length is $\int_a^b \sqrt{(x')^2+(y')^2}\,dt$. Slope is $dy/dx = y'(t)/x'(t)$.

Polar coordinates: $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2+y^2$. Area enclosed by $r = f(\theta)$, $a \le \theta \le b$ is

$$\int_a^b \frac{1}{2}(f(\theta))^2\,d\theta.$$

Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1} = a/(1-r)$ if $|r| < 1$.

Divergence Test: If $\lim_{n\to\infty} a_n$ does not exist or is not 0 .

Integral Test: For $f(x) \ge 0$ continuous and decreasing, $a_n = f(n)$ then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges

P-series: $\sum_{n=1}^{\infty} 1/n^p$ converges if and only if $p > 1$.

Absolute Convergence $\sum_{n=0}^{\infty} |a_n| < \infty$ implies $\sum_{n=0}^{\infty} a_n$ converges

Alternating Series Test If $b_n \ge 0$ is a decreasing sequence and $\lim_{n\to\infty} b_n = 0$. then $\sum_{n=1}^{\infty}(-1)^n b_n$ converges.

Ratio Test. If $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$ then $L < 1$ implies absolute convergence and $L > 1$ implies divergence.

Root Test. The root test is the same as the ratio test above but this time $L = \lim_{n\to\infty}|a_n|^{1/n}$.

Taylor Polynomial of degree n**:** $T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^n(a)}{n!}(x-a)^n$

Remainder Formula: $f(x) = T_n(x) + R_n(x)$ where $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$ for some z, between a and x.

Distance The distance from $P(x_1,y_1,z_1)$ to $Q(x_2,y_2,z_2)$ is $|\vec{PQ}| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

Dot Product: $\text{comp}_{\vec a}\vec b = \frac{\vec a\cdot\vec b}{|\vec a|}$ $\text{proj}_{\vec a}\vec b = \frac{\vec a\cdot\vec b}{|\vec a|^2}\vec a$ $\vec a\cdot\vec b = |\vec a||\vec b|\cos\theta$ where θ is the angle between vectors.

Cross Product: $\vec a\times\vec b$ is vector whose direction is perpendicular to $\vec a$ and $\vec b$ (right hand rule) and the length is the area of the parallelogram determined by $\vec a$ and $\vec b$.

Symmetric and Paramtric Equations of the line A line through $P(x_0,y_0,z_0)$ and in the direction $\vec v = a\vec i + b\vec j + c\vec k$ has parametric equations $x = x_0+ta$, $y = y_0+tb$, $z = z_0+tc$ and symmetric equations

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

Plane The plane with normal $a\vec i + b\vec j + c\vec k$ and passing through $P(x_0,y_0,z_0)$ has equation $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$