May 7, 2010 REVIEW Name ________________

A non graphing calculator is permitted but no calculator is needed. A formula sheet is also allowed.

Common Assessment Questions: 3, 4, 6, 10 , 13 (Marked with *)

1. Compute the limit.

(10)

   (a) $\displaystyle\lim_{x\to 0}\frac{\sin^{-1} 2x}{3x}$

   (b) $\displaystyle\lim_{x\to\infty}\frac{(\ln x)^2}{x}$

2. * Evaluate the integral.

(31)

   (a) $\displaystyle\int xe^{2x}\,dx$

   (b) $\displaystyle\int \sqrt{4-x^2}\,dx$

   (c) $\displaystyle\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx$

3. * Set up, but do NOT evaluate an integral that represents the volume of the solid obtained by rotating the region bounded by the curve $y = \ln x$, and two lines, $y = 0$ and $x = 3$ about the y-axis. Suggestion: Sketch the region.

(10)

4. Determine whether the integral is convergent or divergent. If it is convergent then evaluate it.

(10)

$$\int_1^\infty \frac{2x}{1+x^4}\,dx$$

5. * Find an equation for the tangent line to the curve $x = te^t$, $y = t + e^t$, at the point corresponding to $t = 0$.

(10)

6. Determine whether the series converges or diverges. If it converges then find the sum.

(12)

$$7-\frac{14}{3}+\frac{28}{9}-\frac{56}{27}+\frac{112}{81}-\frac{224}{243}+\dots$$

7. (a) Sketch the curve with polar equation $r = \cos 2\theta$.

(16)

   (b) Set up, but do NOT evaluate an integral that represents the area enclosed by one loop of the curve $r = \cos 2\theta$ of part (a).

8. * Determine whether the series is convergent or divergent. Explain your reasoning.

(16)

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{2}{\sqrt{n^3+5}}$

(b) $\sum_{n=1}^{\infty} \frac{5^n}{n(n!)}$

9. Determine whether the series is absolutely convergent, conditionally convergent or divergent. Explain your reasoning.

(18)

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n(2n+1)}{n+3}$

10. Find the interval of convergence of the series. (You need not check convergence at the endpoints.)

(12)

$$\sum_{n=2}^{\infty} \frac{n-1}{2^n}(x-1)^n$$

11. * Find the Taylor polynomial $T_n(x)$ of $f(x) = \sin x$ at $a = \pi/2$ (that is in powers of $x - \pi/2$) of degree $n = 4$.

(12)

12. Recall that the Maclaurin series expansion for $\cos x$ is

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}.$$

Evaluate the indefinite integral below as an infinite power series (in powers of x.)

(12)

$$\int \cos(3x^2)\, dx$$

(6) 13. Describe in words the region of $\mathbb{R}^3$ represented by $x^2 + y^2 + z^2 < 4z$.

(7) 14. Find the scalar and vector projection of $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ onto $\vec{a} = -\vec{i} + \vec{j} - \vec{k}$.

15. (a) Find a unit vector orthogonal to the plane through $P(0,1,0)$, $Q(2,2,1)$ and
(8) $R(3,1,-1)$, and (b) find the area of triangle PQR.

16. Find an equation for the plane through (2,-3,4) that contains the line $x = 4 - 2t$,
(8) $y = 1 + t$ and $z = 3t$.