2 Pages | **Quiz 9B**, Math 1850 | Section 012
11-21-2013 | Solutions | Name ____________________

(9) 1. Find the most general antiderivative or indefinite integral.

(a) $\displaystyle\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt$

The general antiderivative is

$$\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt = \int t^{-1/2}+t^{-3/2}\,dt = 2t^{1/2}+\frac{1}{-1/2}t^{-1/2}+C = 2t^{1/2}-2t^{-1/2}+C$$

Check by differentiation:

$$\frac{d}{dt}\left[2t^{1/2}-2t^{-1/2}\right] = 2\frac{1}{2}t^{-1/2}-2\frac{-1}{2}t^{-3/2} = t^{-1/2}+t^{-3/2}$$

It checks.

(b) $\displaystyle\int (e^{3x}+5e^{-x})\,dx$

The general antiderivative is

$$\int (e^{3x}+5e^{-x})\,dx = \frac{1}{3}e^{3x}-5e^{-x}+C$$

Check by differentiation:

$$\frac{d}{dx}\left[\frac{1}{3}e^{3x}-5e^{-x}\right] = \frac{1}{3}e^{3x}3-5e^{-x}(-1) = e^{3x}+5e^{-x}$$

It checks.

(c) $\displaystyle\int (\sin 2x-\csc^2 x)\,dx$

The general antiderivative is

$$\int (\sin 2x-\csc^2 x)\,dx = -\frac{1}{2}\cos 2x+\cot x+C$$

Check by differentiation:

$$\frac{d}{dx}\left[-\frac{1}{2}\cos 2x+\cot x\right] = -\frac{1}{2}(-\sin 2x(2))-(\csc x)^2 = \sin 2x-(\csc x)^2$$

It checks

(4) 2. Solve the initial value problem $\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{x}}$, $y(4)=0$.

Certainly y is an antiderivative of $1/(2\sqrt{x})=(1/2)x^{-1/2}$ so that

$$y=\int \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2}\frac{1}{1/2}x^{1/2}+C = x^{1/2}+C$$

But we further know that $y(4) = 0$ so that

$$0 = y(4) = 4^{1/2} + C = 2 + C$$

from which we derive $C = -2$ so that $y = x^{1/2} - 2$. Check by differentiation:

$$y' = \frac{1}{2}x^{1/2}.$$

It checks.

3. Using four rectangles whose height is given by the value of the function at the midpoint of the rectangle's base (*the midpoint rule*), estimate the area under the
(7) graph of

$$f(x) = \frac{1}{x} \quad \text{between } x = 1 \text{ and } x = 5.$$

We divide the interval [1,5] into $n = 4$ equal pieces, each of length $\Delta x = (5 - 1)/4 = 1$ Since we are asked for *the midpoint rule* which means using the values of $f(x) = 1/x$ at the midpoints of the subintervals. One sees that the midpoints are 3/2, 5/2, 7/2, 9/2. The Riemann sum, using the midpoint rule is

$$R(f, \mathcal{P}) = f(3/2)\Delta x + f(5/2)\Delta x + f(7/2)\Delta x + f(9/2)\Delta x = \frac{2}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9} = \frac{496}{315}$$