2 Pages | **Quiz 9A**, Math 1850 | Section 011
11-21-2013 | Solutions | Name ____________________

(9) 1. Find the most general antiderivative or indefinite integral.

(a) $\int 2x(1-x^{-3})\,dx$

The general antiderivative is

$$\int 2x - 2x^{-2}\,dx = x^2 - 2\,\frac{1}{-1}x^{-1} + C = x^2 + 2x^{-1} + C$$

Check by differentiation.

$$\frac{d}{dx}\left[x^2 + 2x^{-1}\right] = 2x + 2(-1)x^{-2} = 2x - 2x^{-2}$$

It checks.

(b) $\int (2e^x - 3e^{-2x})\,dx$

The general antiderivative is

$$\int (2e^x - 3e^{-2x})\,dx = 2e^x - 3\frac{1}{-2}e^{-2x} + C = 2e^x + \frac{3}{2}e^{-2x} + C$$

Check by differentiation:

$$\frac{d}{dx}\left[2e^x + \frac{3}{2}e^{-2x}\right] = 2e^x + \frac{3}{2}\,e^{-2x}(-2) = 2e^x - 3e^{-2x}$$

It checks.

(c) $\int (\sin 2x - \csc^2 x)\,dx$

The general antiderivative is

$$\int (\sin 2x - \csc^2 x)\,dx = -\frac{1}{2}\cos 2x + \cot x + C$$

Check by differentiation:

$$\frac{d}{dx}\left[-\frac{1}{2}\cos 2x + \cot x\right] = -\frac{1}{2}(-\sin 2x(2)) - (\csc x)^2 = \sin 2x - (\csc x)^2$$

It checks

(4) 2. Solve the initial value problem.

$$\frac{dy}{dx} = \frac{1}{x^2} + x,\ x > 0,\ y(2) = 1$$

We see y is an anti-derivative

$$y = \int \frac{1}{x^2} + x\,dx = \int x^{-2} + x\,dx = \frac{1}{-1}x^{-1} + \frac{1}{2}x^2 + C = -\frac{1}{x} + \frac{1}{2}x^2 + C$$

but we further know that $y(2) = 1$ so that

$$1 = y(2) = -\frac{1}{2} + \frac{1}{2}2^2 + C = \frac{3}{2} + C$$

so that $C = -1/2$ and $y = -\frac{1}{x} + \frac{1}{2}x^2 - 1/2$. Check by differentiation:

$$y' = -(-x^{-2}) + \frac{1}{2}2x = x^{-2} + x$$

and $y(2) = 1$.

3. Use finite approximations to estimate the area under the graph of the function using an upper sum with four rectangles of equal width.

$$f(x) = 1/x \quad \text{between } x = 1 \text{ and } x = 5$$

(7)

We divide the interval [1,5] into $n = 4$ equal pieces, each of length $\Delta x = (5-1)/4 = 1$. Since we are asked for the upper sum we want the maximum value of $f(x) = 1/x$ intervals. One sees that the maximum value occurs at the left endpoint because $f(x) = 1/x$ is decreasing. The upper Riemann sum is

$$U(f, \mathcal{P}) = f(1)\Delta x + f(2)\Delta x + f(3)\Delta x + f(4)\Delta x = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$$