2 Pages **Quiz 8B**, Math 1850 Section 012
11-14-2013 Solutions Name ____________

(9) 1. For the rational function $y = \dfrac{x^2}{x^2-1}$ (Suggestion: $y = \dfrac{x^2}{x^2-1} = 1 + \dfrac{1}{x^2-1}$.)

(a) Find all horizontal and vertical asymptotes

(b) Find all critical points.

(c) Find the intervals of increase and decrease.

(d) Sketch the graph of the function.

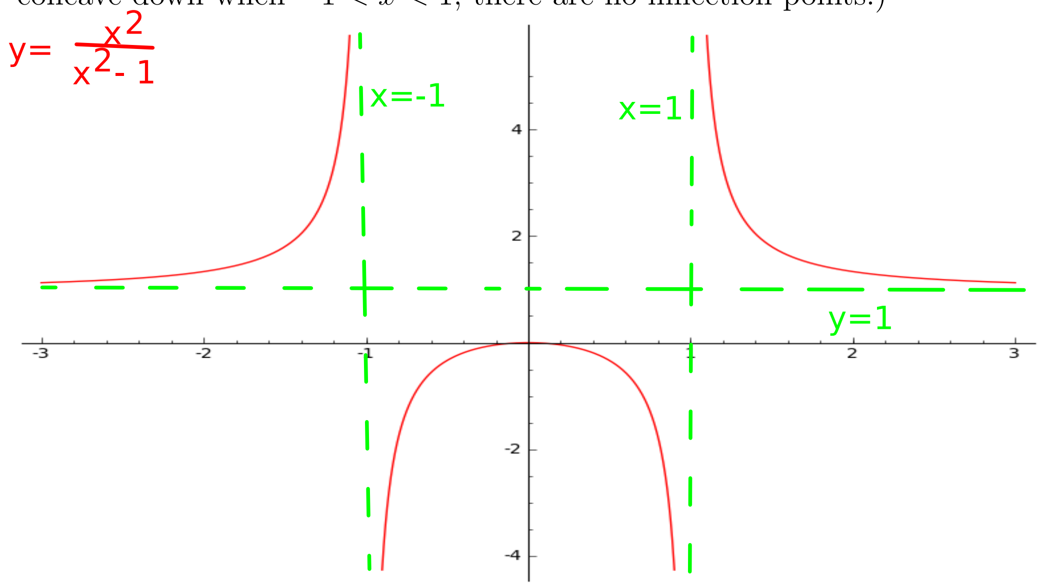
Here $y = \dfrac{x^2}{x^2-1} = \dfrac{x^2}{(x-1)(x+1)}$ so that the vertical asymptotes are $x = 1$ and $x = -1$. Also $\lim_{x\to\pm\infty} x^2/(x^2-1) = 1$ and so $y = 1$ is the horizontal asymptote at $x = \pm\infty$. Compute

$$y' = \frac{-2x}{(x^2-1)^2}$$

If we set $y' = 0$ then there is only one critical point at $x = 0$ but $x = 1$ and $x = -1$ are end points of the domain. The intervals of increase and decrease are:

Intervals	Evaluate $y'(x)$	Increasing or Decreasing?
$-\infty < x < -1$	$y'(-2) = 4/9 > 0$	Increasing
$-1 < x < 0$	$y'(-1/2) = 16/9$	Increasing
$0 < x < 1$	$y'(1/2) = -9/16$	Decreasing
$1 < x < \infty$	$y'(2) = -4/3$	Decreasing

When graphing we include the asymptotes. Also note the function is even and so the graph is symmetric about the y-axis. (The second derivative is NOT required but $y'' = (6x^2+2)(x^2-1)^{-3}$ and so the curve is concave up when $|x| > 1$ and concave down when $-1 < x < 1$; there are no inflection points.)



(3) 2. Find the limits. Use l'Hôpital's Rule if appropriate. $\lim_{t \to 1} \dfrac{t^3 - 1}{4t^3 - t - 3}$

Check that l'Hôpital's Rule does applies (that is the top and bottom of the fraction go to 0.)

$$\lim_{t \to 1} \frac{t^3 - 1}{4t^3 - t - 3} \left(= \frac{0}{0}\right) = \lim_{t \to 1} \frac{3t^2}{12t - 1} = \frac{3}{11}$$

3. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single strand electric fence. With 800 m of wire at your

(8) disposal what is the largest area you can enclose, and what are its dimensions?

Draw a picture of the rectangular plot along a river. Let the plot be x units long and y units wide. We want to maximize the area $A = xy$. We know there is 800 m of fence so that $x + 2y = 800$ or $x = 800 - 2y$. Eliminating x we have $A = (800 - 2y)y = 800y - 2y^2$. We further know that $0 \leq y \leq 400$. Apply the closed interval method. First find the critical points. $A' = 800 - 4y$ Set $A' = 0$ so that $800 = 4y$ and $y = 200$. Since A' is defined everywhere this is the only critical point.

Critical and End Points	Evaluate $A(x)$	Absolute Max/Min?
0	$A(0) = 0$	Absolute Min
200	$A(200) = 80,000$	Absolute Max
400	$A(400) = 0$	Absolute Min

Therefore the absolute maximum area is 80,000 square meters and the rectangular plot will be 200×400. It is also possible to use the first derivatve test and show that of $0 < y < 200$ A is increasing ($A'(y) > 0$) and A is decreasing for $200 < y < 400$ so the $y = 200$ is a local max and since there are no other critical points it is the absolute max. (It is required that you show your reasoning.)