2 Pages **Quiz 8A**, Math 1850 Section 011
11-14-2013 Solutions Name ____________

(9) 1. For the rational function $y = \dfrac{1}{x^2 - 1}$

(a) Find all horizontal and vertical asymptotes

(b) Find all critical points.

(c) Find the intervals of increase and decrease.

(d) Sketch the graph of the function.

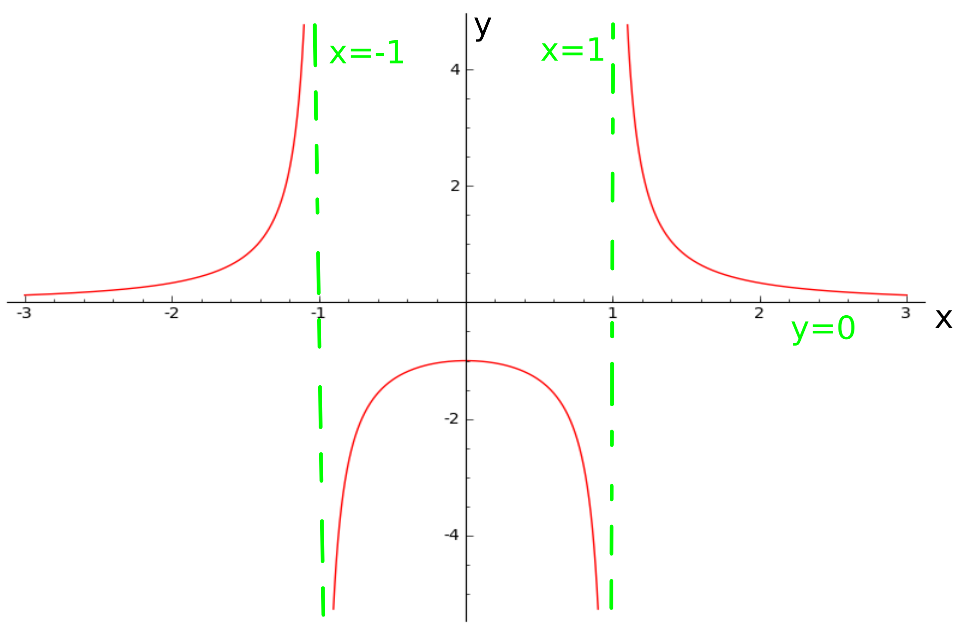
Here $y = \dfrac{1}{x^2 - 1} = \dfrac{1}{(x-1)(x+1)}$ so that the vertical asymptotes are $x = 1$ and $x = -1$. Also $\lim_{x \to \pm\infty} 1/(x^2 - 1) = 0$ and so $y = 0$ is the horizontal asymptote at $x = \pm\infty$. Compute

$$y' = \frac{-2x}{(x^2 - 1)^2}$$

If we set $y' = 0$ then there is only one critical point at $x = 0$ but $x = 1$ and $x = -1$ are end points of the domain. The intervals of increase and decrease are:

Intervals	Evaluate y'	Increasing or Decreasing?
$-\infty < x < -1$	$y'(-2) = 4/3 > 0$	Increasing
$-1 < x < 0$	$y'(-1/2) = 16/9$	Increasing
$0 < x < 1$	$y'(1/2) = -16/9$	Decreasing
$1 < x < \infty$	$y'(2) = -4/9$	Decreasing

When graphing we include the asymptotes. Also note the function is even and so the graph is symmetric about the y-axis. (The second derivative is NOT required but $y'' = (6x^2 + 2)(x^2 - 1)^{-3}$ and so the curve is concave up when $|x| > 1$ and concave down when $-1 < x < 1$; there are no inflection points.)



(3) 2. Find the limit. Use l'Hôpital's Rule if appropriate. $\lim_{t \to -3} \frac{t^3 - 4t + 15}{t^2 - t - 12}$

Check that l'Hôpital's Rule applies (that is the top and bottom of the fraction both go to 0)

$$\lim_{t \to -3} \frac{t^3 - 4t + 15}{t^2 - t - 12} \left(= \frac{0}{0}\right) \lim_{t \to -3} \frac{3t^2 - 4}{2t - 1} = \frac{27 - 4}{-6 - 1} = -\frac{23}{7}$$

3. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have and what are its
(8) dimensions.

We are trying to maximize the area of a rectangle which is base times height. From the picture we see that the base is $2x$ and the height is $12 - x^2$ so that the area is

$$A = 2x(12 - x^2) = 24x - 2x^3,$$

We further see that $0 \leq x \leq \sqrt{12}$ because the parabola intersects the x-axis at $\pm\sqrt{12}$. We can use the closed interval method. Compute $A' = 24 - 6x^2$ so that $A' = 0$ implies $24 - 6x^2 = 0$ or $x = \pm 2$ but $x \geq 0$ and so $x = 2$ is the only relevant critical point. The endpoints are $x = 0$ and $x = \sqrt{12}$. Evaluate.

Critical and End Points	Evaluate $A(x)$	Absolute Max/Min?
0	$A(0) = 0$	Absolute Min
2	$A(2) = 32$	Absolute Max
$\sqrt{12}$	$A(\sqrt{12}) = 0$	Absolute Min

The largest the rectangle can be is 4 by 8 with area 32. The problem can be done also using the first derivative test and show that for $0 < x < 2$, $A(x)$ is increasing and for $2 < x < \sqrt{12}$ it is decreasing so that $x = 2$ must be a local max and since it is the only critical point it must be the absolute max. (It is required that you show your reasoning.)